Appendix A

Calculation of the Fixed Point

The objective of this section is to describe the method of determining the fixed points of the static aeroelastic system whose aerodynamic forces are given by the Kirchhoff's theory.

The fixed points of the static aeroelastic system are defined in section 4.2 as the point where $\mathbf{x}' = \mathbf{0}$ and are the solutions of the algebraic equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$. It can also be deduced from this definition that the fixed points of the full and simplified aeroelastic systems must coincide. Moreover, the Kirchhoff's theory represents the LB-model under the static conditions [8].

First, at the fixed point, the values of the airfoil geometric pitch rate x_{14} and the vertical velocity x_{16} must be zero. This will be used to simplify the uncoupled equations of motion of a 2-DOF airfoil given by Fung [10] whose solutions represent the position of the airfoil static equilibrium. The equilibrium geometric angle of incidence x_{13}, is the solution of the nonlinear equation

$$\frac{\mu\pi r_\alpha^2}{2U^{*2}}(x_{13} - \theta_0) = \frac{\left(\frac{1}{2} + a_h\right)}{2} C_{N_{static}}(x_{13}) + C_{M_{static}}(x_{13}) \tag{A.1}$$

With the equilibrium angle of incidence, the corresponding nondimensional vertical displacement x_{15} is the solution of the equation

$$\left(\frac{\bar{\omega}}{U^*}\right)^2 x_{15} = -\frac{1}{\pi\mu} C_{L_{static}}(x_{13}) \tag{A.2}$$

The expressions of the aerodynamic terms in equations (A.1) and (A.2) are given in section 2.2. The effective angle of incidence and the pitch rate under the static condition are given by $\alpha = x_{13}$ and $q = 0$, respectively. The remaining states of the fixed point are given by

$$x_1 = -\frac{x_{13}}{a_{11}} \tag{A.3}$$

$$x_2 = -\frac{x_{13}}{a_{22}} \tag{A.4}$$

$$x_3 = -\frac{x_{13}}{a_{33}} \tag{A.5}$$

$$x_4 = 0 \tag{A.6}$$

$$x_5 = -\frac{x_{13}}{a_{55}} \tag{A.7}$$

$$x_6 = -\frac{x_{13}}{a_{66}} \tag{A.8}$$

$$x_7 = 0 \tag{A.9}$$

$$x_8 = 0 \tag{A.10}$$

$$x_9 = c_{11}x_1 + c_{12}x_2c_{13}x_3 + \frac{4}{M}x_{13} \tag{A.11}$$

$$x_{10} = \begin{cases} 1 - 0.30exp\left(\frac{|\frac{x_9}{C_{N\alpha}}|-\alpha_1}{S_1}\right) & \text{if } |\frac{x_9}{C_{N\alpha}}| \leq \alpha_1 \\ 0.04 + 0.66exp\left(\frac{\alpha_1-|\frac{x_9}{C_{N\alpha}}|}{S_2}\right) & \text{if } |\frac{x_9}{C_{N\alpha}}| > \alpha_1 \end{cases} \tag{A.12}$$

$$x_{11} = 0 \tag{A.13}$$

$$x_{12} = \begin{cases} 1 - 0.30exp\left(\frac{|x_{13}|-\alpha_1}{S_1}\right) & \text{if } |x_{13}| \leq \alpha_1 \\ 0.04 + 0.66exp\left(\frac{\alpha_1-|x_{13}|}{S_2}\right) & \text{if } |x_{13}| > \alpha_1 \end{cases} \tag{A.14}$$

Note that $\frac{x_9}{C_{N\alpha}} = x_{13}$ at the fixed point, hence the switches in equations (A.12) and (A.14) will be activated simultaneously. This switch also relates to the instance that the condition $x_{10} = 0.7$ is satisfied.