Lecture 10 Aeroelastic System

We saw in the previous lecture that the unsteady aerodynamic forces could be computed with a dynamic stall written in the state space form. This particular dynamic stall model is a 12-state (or 12-dimensional) system, meaning the model is written as

$$\mathbf{x}' = \mathbf{f}(\mathbf{x})$$
$$\begin{Bmatrix} C_L \\ C_M \end{Bmatrix} = \mathbf{g}(\mathbf{x}) \qquad \text{[EQN 1]}$$

where $\mathbf{x}'$ represents differentiation with respect to nondimensional time, τ
$\mathbf{x}$ is a vector of state variables, $\mathbf{x} = \{x_1 \quad x_2 \quad \cdots \quad x_{12}\}$

Now, recall that we are studying an aeroelastic system, which focuses on the interaction between aerodynamic forces and structural motion. The aerodynamic forces have been defined. Generally, the structural motion is defined by the equations of motion of a two degree-of-freedom aerofoil such as

$$\begin{aligned} m\ddot{h} + S_\alpha \ddot{\alpha} + C_h \dot{h} + K_h h &= -L(t) \\ S\ddot{h} + I_\alpha \ddot{\alpha} + C_\alpha \dot{\alpha} + K_\alpha \alpha &= M(t) \end{aligned} \qquad \text{[EQN 2]}$$

where the symbols $m, S_\alpha, C_h, C_\alpha, I_\alpha$ are airfoil mass, airfoil static moment about the elastic axis, damping coefficient in pitch, torsion damping constant and wing mass moment of inertia respectively. $L(t), M(t)$ are total lift forces and moments acting on the airfoil respectively.

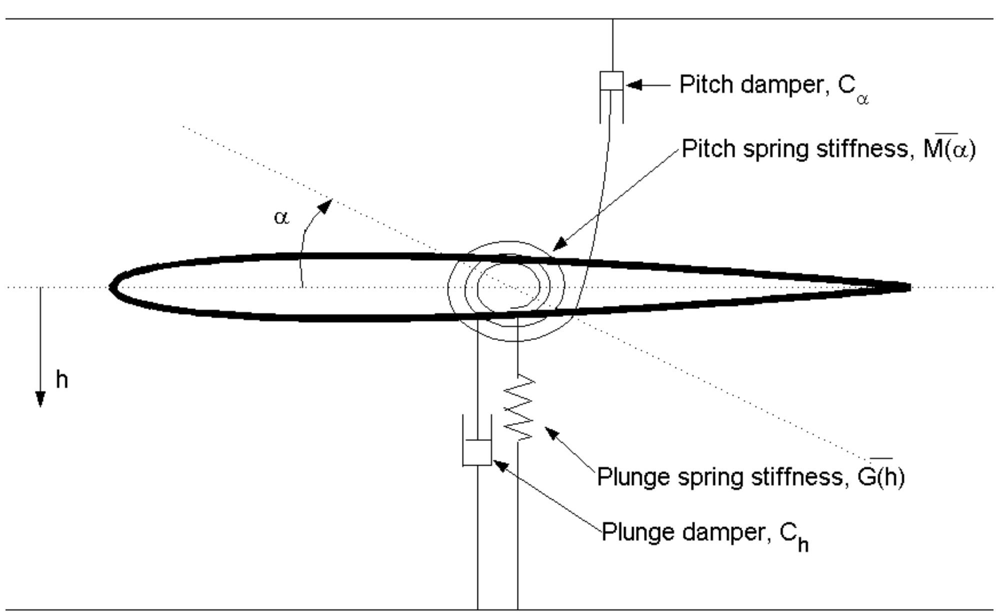


Figure 1 Simple two degree-of-freedom spring damper system

Nondimensional Equations of Motion

It is a common practice to write the equations of motion in non-dimensional form. Hence equations [2] can be rewritten as

$$\xi'' + x_\alpha \alpha'' + 2\varsigma_\xi \frac{\bar{\omega}}{U^*}\xi' + \left(\frac{\bar{\omega}}{U^*}\right)^2 G(\xi) = -\frac{1}{\pi\mu} C_L(\tau) + \frac{P(\tau)b}{mU^2} \qquad \text{[EQN 3a]}$$

$$\frac{x_\alpha}{r_\alpha^2}\xi'' + \alpha'' + 2\frac{\varsigma_\alpha}{U^*}\alpha' + \frac{1}{U^{*2}} M(\alpha) = \frac{2}{\pi\mu r_\alpha^2} C_M(\tau) + \frac{Q(\tau)}{mU^2 r_\alpha^2} \qquad \text{[EQN 3b]}$$

where

$$\xi = \frac{h}{b} \qquad \omega_\alpha = \sqrt{\frac{K_\alpha}{I_\alpha}}$$

$$K_\xi = K_h \qquad r_\alpha = \sqrt{\frac{I_b}{mb^2}}$$

$$x_\alpha = \frac{S}{bm} \qquad \varsigma_\xi = \frac{C_h}{2\sqrt{mK_h}}$$

$$\omega_\xi = \sqrt{\frac{K_\xi}{m}} \qquad \varsigma_\varepsilon = \frac{C_\alpha}{2\sqrt{I_\alpha K_\alpha}}$$

Non-dimensional velocity is defined as

$$U^* = \frac{U}{b\omega_\alpha}$$

Uncoupled plunging and pitching modes natural frequencies ratio is given as

$$\bar{\omega} = \frac{\omega_\xi}{\omega_\alpha}$$

Prime denotes differentiation with respect to non-dimensional time defined as

$$\tau = \frac{Ut}{b}$$

$C_L(\tau)$ and $C_M(\tau)$ are non-linear aerodynamic lift and pitching moment coefficients defined by the aerodynamic model. $P(\tau)$ and $Q(\tau)$ are externally applied forces and torque respectively.

Recall that, second order ODEs such as equations 3a and 3b can be rewritten as a system of four first order ODEs. Hence, the nondimensional equations of motion can be given by

$$\alpha'' = \frac{1}{\left[\left(\frac{x_\alpha}{r_\alpha}\right)^2 - 1\right]}\left[\begin{array}{l} 2\frac{\varsigma_\alpha}{U^*}\alpha' + \frac{1}{U^{*2}}M(\alpha) - 2\varsigma_\xi\left(\frac{\bar{\omega}}{U^*}\right)\left(\frac{x_\alpha}{r_\alpha^2}\right)\xi' - \left(\frac{x_\alpha}{r_\alpha^2}\right)\left(\frac{\bar{\omega}}{U^*}\right)^2 G(\xi) \\ -\frac{2}{\pi\mu r_\alpha^2}C_M(\tau) - \frac{x_\alpha}{\pi\mu r_\alpha^2}C_L(\tau) - \frac{Q(\tau)}{mU^2 r_\alpha^2} + \left(\frac{x_\alpha}{r_\alpha^2}\right)\frac{b}{mU^2}P(\tau) \end{array}\right]$$

[EQN 4a]

$$\xi'' = \frac{1}{\left[\left(\frac{x_\alpha}{r_\alpha}\right)^2 - 1\right]}\left[\begin{array}{l} 2\varsigma_\xi\left(\frac{\bar{\omega}}{U^*}\right)\xi' - 2\frac{\varsigma_\alpha}{U^*}x_\alpha\alpha' - \frac{x_\alpha}{U^{*2}}M(\alpha) + \left(\frac{\bar{\omega}}{U^*}\right)^2 G(\xi) \\ +\frac{2x_\alpha}{\pi\mu r_\alpha^2}C_M(\tau) + \frac{1}{\pi\mu}C_L(\tau) + \frac{x_\alpha}{mU^2 r_\alpha^2}Q(\tau) - \frac{b}{mU^2}P(\tau) \end{array}\right]$$

[EQN 4b]

Letting $\mathbf{z} = \{z_1 \quad z_2 \quad z_3 \quad z_4\}^T = \{\alpha \quad \alpha' \quad \xi \quad \xi'\}^T$, we obtain

$$\mathbf{z}' = \mathbf{f}(\mathbf{z}, C_L, C_M) \qquad \text{[EQN 5]}$$

Formation of Aeroelastic System

You may have noticed that both equation 5 and equation 1 are systems of first order ODEs in the nondimensional time domain. It is, therefore, possible to combine the structural model with the aerodynamic model by merging these two sets of equations together, i.e.

$$\mathbf{X}' = \mathbf{F}(\mathbf{X}, C_L, C_M)$$
$$\begin{Bmatrix} C_L \\ C_M \end{Bmatrix} = \mathbf{G}(\mathbf{X}) \qquad \text{[EQN 6]}$$

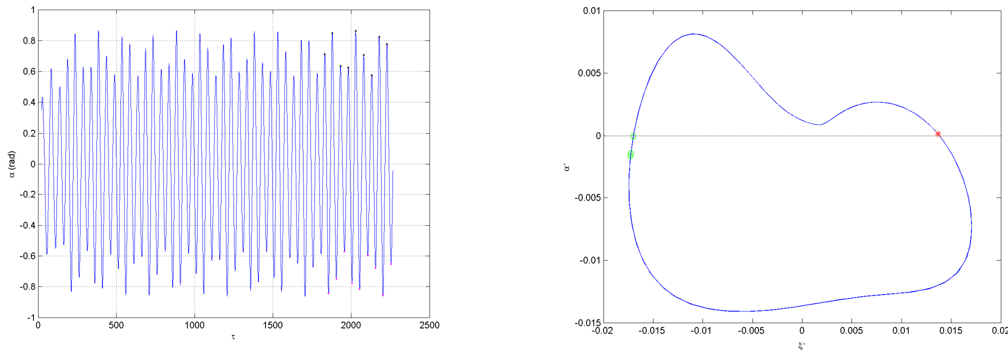
where $\mathbf{X} = \{x_1 \quad x_2 \quad \cdots \quad x_{12} \quad z_1 \quad \cdots \quad z_4\}^T$

Numerical Solutions of Aeroelastic Flutter

As you can clearly see, the reason for using the state-space dynamic stall model formulation is that it can be coupled with the structural model to form a complete aeroelastic model as shown in equation 6.

The solutions of the aeroelastic system can be found by solving (integrating) the system of ODEs in equation 6. One possible way to achieve this is to use MATLAB's ode45 function which utilizes the Runge-Kutta-Fehlberg integration algorithm. The software will compute the solutions provided that the user has selected other parameters of the system. This is a very common method to determining the stability of the aeroelastic system.

Solutions of the aeroelastic system are always functions of time and they may be presented in a number of ways such as time history, or phase portraits.



Time history *Phase portrait*

Figure 2 Examples of numerical representations of aeroelastic system solutions

Recall that, flutter prediction is the most important part of dynamic aeroelasticity because it is the point where the aeroelastic system loses its stability. Hence, we can use the tool called 'bifurcation diagram' to determine the critical point where flutter first occurs.

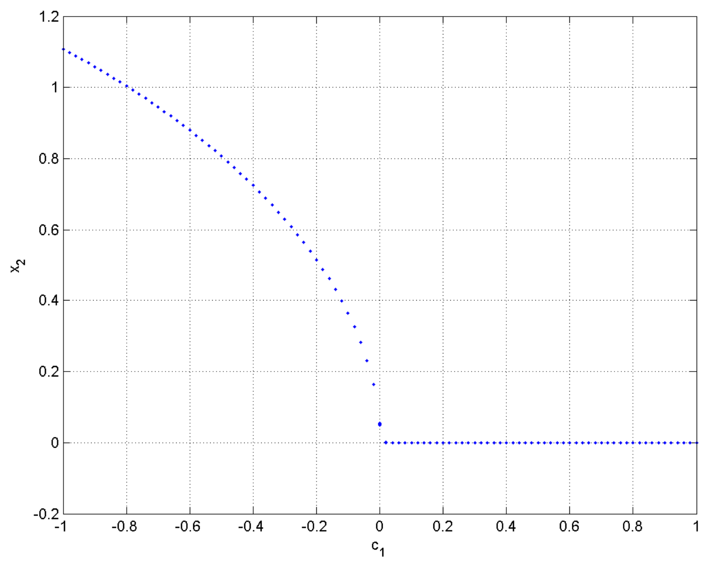


Figure 3 Example of bifurcation diagram of an aeroelastic system

Analytical Solution of Aeroelastic Flutter

The aeroelastic flutter can also be determined analytically too. This involves an analysis of the stability of the fixed point of the system. In short, a stable aeroelastic system (no flutter) will have a stable fixed point while during flutter this fixed point is unstable.

Please refer to supplementary lecture notes 10a and 10b for further information.