Lecture 5 – Flutter: Part 2 Stability Analysis

Previously, we derived the equations of motion of a two degree-of-freedom aerofoil under a steady flow which are given by

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[EQN.1]

____

Rewriting equation 1 in matrix form, we obtain

____ [EQN.2]

where

<table>
<tr><td>$\mathbf{q} = \begin{Bmatrix} h \\ \alpha \end{Bmatrix}$</td><td>denotes the generalised coordinates</td></tr>
<tr><td>$\mathbf{M} = \begin{bmatrix} m & mx_{CG} \\ mx_{CG} & I_\alpha \end{bmatrix}$</td><td>denotes the mass matrix</td></tr>
<tr><td>$\mathbf{K} = \begin{bmatrix} k_h & 0 \\ 0 & k_\alpha \end{bmatrix}$</td><td rowspan="2">$(\mathbf{K} + q\mathbf{A})$ is the stiffness matrix</td></tr>
<tr><td>$\mathbf{A} = S\begin{bmatrix} 0 & C_{L\alpha} \\ 0 & x_{ac}C_{L\alpha} \end{bmatrix}$</td></tr>
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Solutions of Equations of Motion

Normally, we know the behaviour of the aeroelastic system such as the pitch angle and pitch rates at the beginning, i.e. the ____________________________________ are known

______________________________ and ______________________________

The solutions of the equations of motion of the two-dof aeroelastic system, which are a pair of ____, take the form of

____ [EQN.3]

where $\overline{\mathbf{q}}$ represents the mean value of the solutions, note that $\overline{\mathbf{q}} = \mathbf{q}_0$

t denotes the time in seconds

The values for p in equation 3 will now be investigated. Generally, p is a complex number, and the real part of p determines the stability of the system. In short, the system may fall into one of the three categories, which are

1. $Re(p) > 0$. The system is unstable.
2. $Re(p) > 0$. The system is neutrally stable.
3. $Re(p) > 0$. The system is stable.

The stability of the system is schematically shown in Figure 1, which depicts the time histories of the real part of $\mathbf{q}$.

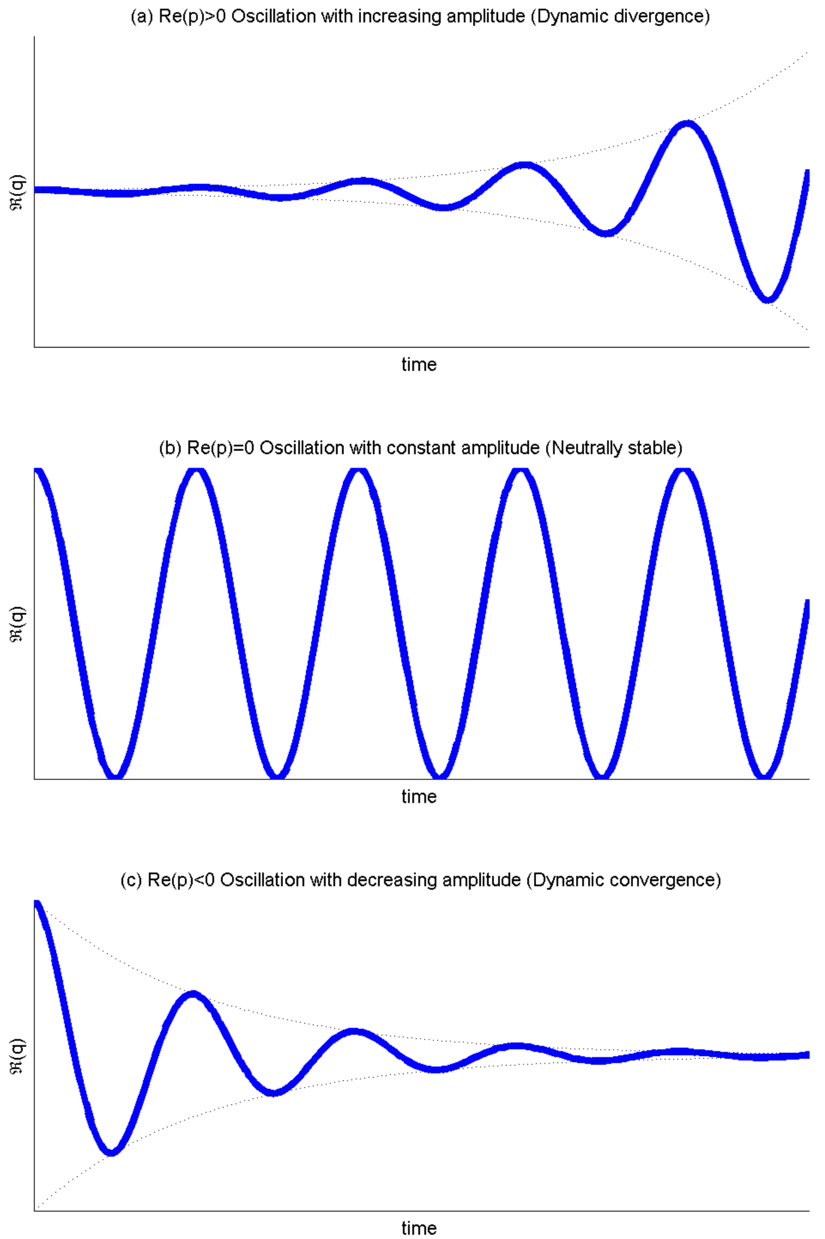

Figure 1 Typical solutions of a two-dof aeroelastic system

Stability Analysis of a Two-DOF Aeroelastic System

From the solutions of the equations of motion given in equation 3, we obtain the following

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Substitute the solutions $\mathbf{q}(t)$ and its second derivatives $\ddot{\mathbf{q}}(t)$ into equation 1 to obtain

____

____ [EQN.4]

The dynamic stability of the aerofoil will be determined by the non-trivial solutions of equation 4, which can be calculated by setting the determinant of the matrix to zero. We obtain

____ [EQN.5]

where

$$A = m\left(I_\alpha - m x_{CG}^2\right)$$
$$B = k_h I_\alpha + k_\alpha m + mqS\left(x_{ac} - x_{CG}\right)C_{L\alpha}$$
$$C = k_h\left(k_\alpha + qSx_{ac}C_{L\alpha}\right)$$

Note that

- The term A is always ____________________. This is because $I_\alpha - m x_{CG}^2 = I_p$, which is the ____ at the centre of mass and it is always positive.
- The term C is also always __________________________.
- Equation 5 is a fourth-order polynomial expression and has _____________ roots.

The solution of equation 5 is, of course, given by the quadratic formula, that is

____

Note that

- Flutter occurs when system moves from ________________________________ to ________________________________.
- Since A and C are both positive, $B > \sqrt{B^2 - 4AC}$

Let us analyse the solution of equation 5 with an ascending values of the dynamic pressure q.

$0 \le q < q_F$		Roots of p^2 are real and negative, therefore p are purely imaginary (zero real part)	The system is neutrally stable.
$q = q_F$		Transition from dynamic stability to dynamic unstability	This point defines the onset of dynamic instability or the flutter point.
$q_F \le q < q_D$		The roots of equation 5 become $p^2 = re^{i\phi}$, hence the four roots are given by $p = \begin{cases} \pm\sqrt{r}e^{i\frac{\phi}{2}} \\ \pm\sqrt{r}e^{i\left(\frac{\phi}{2}+\pi\right)} \end{cases}$	Obviously, the roots with positive real parts cause dynamic unstability, i.e. dynamic divergence

where q_F is the flutter speed, which can be calculated using the expression

$$\left[k_\alpha I_\alpha + k_\alpha m + mq_F S\left(x_{ac} - x_{CG}\right)C_{L\alpha}\right]^2 - 4mk_h\left(I_\alpha - mx_{CG}^2\right)\left(k_\alpha + q_F S x_{ac} C_{L\alpha}\right) = 0$$

This expression is derived by substituting the terms from mass and stiffness matrices into the expression that defines the flutter point, i.e. $B^2 - 4AC = 0$.