Lecture 4 – Flutter: Part 1 Equations of Motion

We will begin the first topic of ____________________________________ with the study of _________________ of an _________________________ in a ___________________.

Dynamic Aeroelasticity

It is the study of __________________ of a structure subjected to aerodynamic forces, in another words, the ____ of the structure. As we know, the aerodynamic forces depend on the structural orientation and deformation, so the equations of motion of such structures involve _______________ the dynamic aerodynamic loadings with structural dynamics.

Flutter Phenomena

Flutter is one of the most dramatic aeroelastic phenomena. One of the most well known engineering catastrophes is the structural failure of the Tacoma Narrows Bridge in 1940. See example here http://www.youtube.com/watch?v=j-zczJXSxnw. (Highly recommended)

Although the application of flutter analysis will apply to most structures that oscillate when exposed to a fluid flow, we will limit our attention to an aerofoil section here.

For simplicity, we will use a typical ____ aerofoil shown in figure 1 in our studies. It is a rigid two dimensional symmetric aerofoil with zero initial twist ($\alpha_r = 0$). Structural damping is neglected here.

Figure 1

L	Lift force
M_{ac}	Pitching moment at the aerodynamic centre
α	Aerofoil twist angle (measured from horizontal and positive nose up)
h	Vertical displacement (measured at elastic axis and positive downward)
U	Free stream velocity
c	Chord length
k_h	Spring stiffness in vertical translation
k_α	Spring stiffness in rotation

Equations of Motion

We will use ____ to determine the equations of motion of the aerofoil in figure 1. Neglecting the damping terms, we obtain

____ [EQN.1] where the terms in equation 1 are

T	Total kinetic energy
U	Total potential energy
$\boldsymbol{q}$	Generalised coordinates, in this case $\mathbf{q} = \begin{Bmatrix} h \\ \alpha \end{Bmatrix}$
$\boldsymbol{Q}$	Generalised forces, in this case $\mathbf{Q} = \begin{Bmatrix} -L \\ M \end{Bmatrix}$
t	Time in seconds

Total Kinetic Energy

Let us consider a ____ of the aerofoil at ________________________ with ____________________________ *(see figure 1).*

Assuming that the twist angle is small, the ___________________________________ of the element is given by

____

Let m` denote the mass per unit length (chord wise), the kinetic energy for one element is given by

____

Hence the total kinetic energy of the aerofoil is given by

____

______________________________________ [EQN.2]

Note that

$\int m\grave{}dx =$	
$\int m\grave{}x^2 dx =$	
$\int m\grave{}x \ dx =$	

Finally, we can rewrite equation 2 to show that the total kinetic energy of the aerofoil is given by

____ [EQN.3]

Total Potential Energy

Only potential energies stored in the springs are included here

____ [EQN.4]

Generalised Forces

The force and moment in the directions of the general coordinates are negative lift and pitching moment respectively. For a symmetric aerofoil at __________________ angles of attack, _____________________________. We normally write aerodynamic forces in their nondimensional forms, hence we obtain

____

[EQN.5]

____

where we use the same nomenclatures as the previous chapters, i.e.

$q = \frac{1}{2}\rho U^2$	Dynamic pressure
S	Wing area (chord x span)
x_{ac}	Distance between CG and elastic axis (positive when elastic axis is ahead of CG)

Equations of Motion of a 2-dof Aerofoil

Finally, differentiate equations 3, 4 and 5 with respect to _________ and ___________, and substitute those expressions into equation 1 to obtain the following equilibrium equations

_______________________________________ [EQN.6]

____

Rewriting the above equations in matrix form, we obtain

_______________________________________ [EQN.7]

with

$\mathbf{M} = \begin{bmatrix} m & mx_{CG} \\ mx_{CG} & I_\alpha \end{bmatrix}$	
$\mathbf{K} = \begin{bmatrix} k_h & 0 \\ 0 & k_\alpha \end{bmatrix}$	
$\mathbf{A} = S\begin{bmatrix} 0 & C_{L\alpha} \\ 0 & x_{ac}C_{L\alpha} \end{bmatrix}$	

We will study the stability of the aeroelastic system by analyzing the ________________ of equation 7 in the next lecture.