Lecture 3 – Divergence of a Lifting Surface

Up to now all our focus has only been on ____ models with flexible supports, hence previous analyses have only given insights into basics of aeroelasticity stability and responses. Now we will extend the analysis further to determine the _____________________ of _______________________________________, i.e. ____.

We will limit our attention to an _________________, ____________________ elastic wing as shown in figure 1. Here, we see a conventional wing configuration which is ___________________ on one end and is ________________ at the other. It behaves like a ______________________________.

Figure 1

Assumptions

We will propose several assumptions to the wing shown in the figure to make calculations possible.

1. The y-axis corresponds to the ______________________________ of the wing, which is also defined as a line of effective _______________________.

2. The only elastic deformation that we will consider is the ____________________ due to ____, i.e. ____.
3. The aerodynamic loads ____ _________________________ along the span of the rectangular planform wing.
4. The aerodynamic lift coefficient at any point along the span is given by _________________________.

Total Applied Moment

Figure 2

All forces and moments shown in figure 2 are loads per unit span, indicated by the ` symbol.

L`	
M_{ac}`	
N	
m	
e	
d	
c	

The loads acting on the wing are

____

____

where the freestream dynamic pressure is $q = \frac{1}{2}\rho U^2$. Finally the ____________________ ____ is given by

____________________________ [EQN.1]

Note that the moment is _______________________.

Torsional Equilibrium

We will use the ___________________________________ analysis to study the divergence of the lifting surface. Recall that the fundamental torsional relationship states that

____________________________ [EQN.2]

where T is the twisting moment about the elastic axis, GJ is the torsional rigidity (stiffness) and θ is the twist angle. Let us now consider a strip element of thickness δy of the wing in figure 1.

Figure 3

From equilibrium condition of the strip element, we obtain

____

______________________________ [EQN.3]

Combining equations 1, 2 and 3, we obtain

____

____

Since our wing is uniform, the stiffness term GJ is constant over the length, therefore we obtain

____

Now let us write the aerodynamic coefficients in terms of pitch angle

____

Rearranging the above expression so that all terms with θ are on the left hand side of the equation

______________________________ [EQN.4]

Equation 4 is a ____. Its solution will be ________________ which represents the ____ along the span of the wing. However, we need more information before we can fully solve equation 4.

The wing in question here is built like a ____, i.e. it has a ______________________root at _______________ and it has ______________ at the wing tip __________________. This information allows us to determine the boundary conditions as

____

____

Solutions of a Second Order ODE

Let us simplify the second order ODE in equation 4 by introducing

____

____

So the ODE can be rewritten as

____________________________ [EQN.5]

The general solution to this linear second order ODE is

____________________________ [EQN.6]

Applying the ______________________________ according to the wing configuration shown in figure 1, we obtain

____

____

Substituting these boundary conditions into equation 6 to obtain the elastic twist distribution

____________________________ [EQN.7]

Divergence of a Lifting Surface

Divergence occurs when ____, i.e. __________________. In the case of a lifting surface whose twist distribution is described by equation 6, divergence occurs when

____________________________ where __________________________

Let us consider the case where_____________, we can determine the divergence dynamic pressure q_D from the expression

____

____