Lecture 9 – Kinetics of a particle Part 2: Work and Energy

The work of a force

Let us first establish the definition of work. A force, denoted by a vector **F**, is said to do _____ on a particle only when the _____
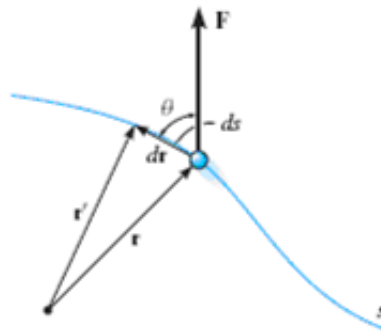
_____.

Figure 1

In the figure, the particle moves along the path from the original position **r** to a new position **r'** . The displacement is given by $d\mathbf{r} = \mathbf{r'} - \mathbf{r}$, whose magnitude is represented by ds. If the angle between $d\mathbf{r}$ and **F** is θ, then the work dU, which is done by the force **F**, is defined by

_____

_____

Note that the unit of work done is called a _____ By definition, 1 joule of work is done when one Newton of force moves one meter along its line of action.

Work of a variable force

A force is said to be _____ when its _____ or

_____ is _____. The total

work done of a variable force can be given by

_____

_____

Work of a constant force

For the case of a force $\mathbf{F}_c$, which of _____ and it acts at a

_____ to the straight line path. The work done

when a particle is displaced from s_1 to s_2 can be determined using

_____

_____

Work of a weight

If we consider a particle of mass $\mathbf{m}$, in a 3D space defined by a set of perpendicular x- y-
and z-axes, its weight must be given by a vector $\mathbf{W} = -W\mathbf{j}$, i.e. its direction is always
directly downwards. Suppose that a particle is displaced in the same 3D space along the
path s from position s_1 to s_2. An intermediate point, the displacement is given by
$d\mathbf{r} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}$. The amount of work done by the weight is given by

$$U_{1-2} = \int \mathbf{F} \cdot d\mathbf{r} = \int_{s_1}^{s_2} \left(-W\mathbf{j}\right) \cdot \left(dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}\right)$$

$$= \int_{y_1}^{y_2} -W\ dy = -W\left(y_2 - y_1\right)$$

_____

Work of a spring force

Unstretched
position, $s = 0$
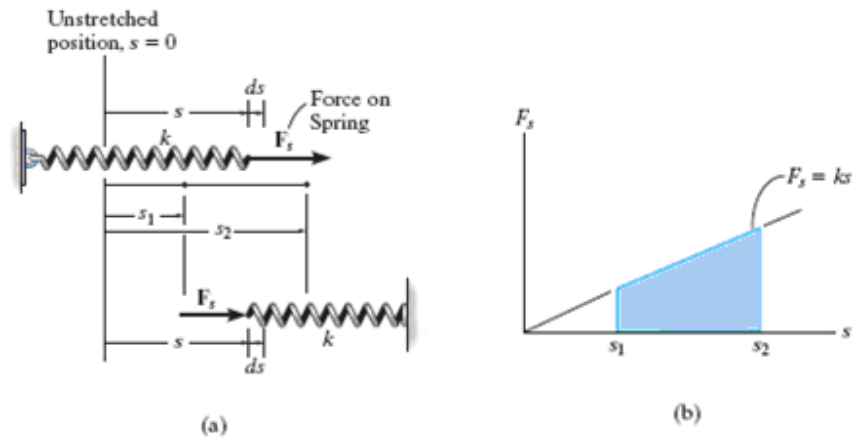

Figure 2

A linear spring of stiffness k provides a restoring force _____when stretched (or compressed) by a displacement s. The force $\mathbf{F}_s$ which stretches (or compresses) the spring is said to have done some work, as _____

_____.

Suppose the spring is stretched or compressed by an applied force $\mathbf{F}_s$ from the original position s_1 to a new position s_2, the total work done during this process is given by

$$U_{1-2} = \int_{s_1}^{s_2} F_s \; ds = \int_{s_1}^{s_2} ks \; ds$$

_____

Note that the work done by the spring can be graphically represented by the area under the force-displacement curve.

Principle of work and energy

Recall the two main types of mechanical energies namely, _____ and
_____ energies. These will be discussed further in their respective
topics in this lecture. One thing of note here is that the SI unit for energy is _____,
which is the same as that used for work done.

Application of principle of work and energy for a system of particles

Consider a particle *P* of mass *m* in figure 3. A resultant force _____
(the resultant force represents a system of external forces) is acting on the particle, so that
it moves along the path *s*, for example from point s_1 to point s_2.

The equation of motion of the particle in the tangential direction is given by _____.
Since the tangential acceleration can be written as _____, the equation
of motion can be rewritten as

$$\Sigma \int_{s_1}^{s_2} F_t \ ds = \int_{v_1}^{v_2} mv \ dv$$

$$U_{1-2} = \frac{1}{2} m(v_2)^2 - \frac{1}{2} m(v_1)^2$$

_____   (EQN. 1)

where _____ represents the _____.

Equation 1 can be rearranged to give _____, which states that the
particle's initial kinetic energy plus the work done by all the forces acting on the particle
as it moves from its initial position to its final position is equal to the particle's final
kinetic energy.

This relationship of work and kinetic energy is particularly useful when one wants to determine the final velocity of the particle.

Equation 1 can also be extended to _____, i.e.

_____

where the summation represent the total work done and kinetic energy of all particles considered in the system.
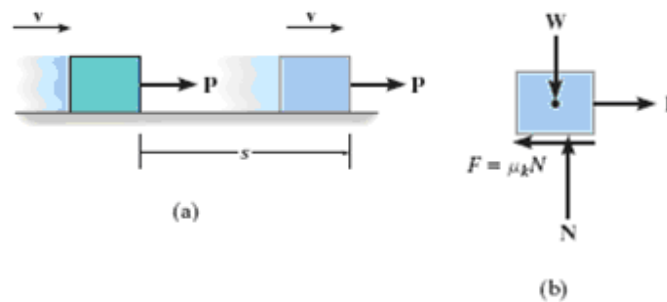
Work of dry friction

Figure 4

The figure shows a block P of mass m being pulled along a horizontal rough surface whose _____.
The initial velocity is given by v_1 and the final velocity is v_2. We can apply equation 1 to obtain

_____

Power

The definition of power states that it is _____

_____. Its mathematical expression is given by

_____

Recalling that the incremental work done is $dU = \mathbf{F} \cdot d\mathbf{r}$, hence it is possible to derive the following.

$$P = \frac{dU}{dt} = \frac{\mathbf{F} \cdot d\mathbf{r}}{dt} = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt}$$

_____

where $\mathbf{v}$ represents the instantaneous velocity of the point which is acted upon by the force $\mathbf{F}$.

The SI unit of power is the _____ and the unit is defined as

_____

The quantity power can be regarded as the capacity to do a certain amount of work in a given time.

Another common unit for power is the horsepower, where 1 horsepower is equivalent to 745.7 W.

Efficiency

Mechanical efficiency of a machine is defined as the _____ of output useful power produced to the input power supplied. The mathematical expression for efficiency is given by

_____

However, if the efficiency is being measured during the same time interval, we can directly evaluate the efficiency by comparing the amount of energy gained and supplied.

_____

Note that some textbooks use the symbol _____ to denote efficiency

Potential energy and conservation of forces

Conservative force

When the work done by a force in moving a particle from one point to another is
_____ , then this force is considered a conservative
force. _____
are examples of conservative forces.

On the other hand, _____
_____, i.e. the longer the path, the
greater the work done. These forces are called nonconservative forces.

Potential energy

The potential energy of a particle comes from _____ measured
relative to a fixed datum. Recall that kinetic energy is related to the _____
of the particle.

Gravitational potential energy

The potential energy of a particle of mass *m* located a distance *y* vertically above an
arbitrary datum is given by

_____

where *g* represents the acceleration due to gravity. It is possible for a particle to have negative potential energy, if its vertical position is below a chosen datum.

Elastic potential energy

When an elastic spring is stretched or compressed by a distance s from the original unstressed position, its elastic potential energy is given by

_____

Note that _____.. This is because the spring always has 'potential' for doing positive work on the particle to bring it back to the original unstretched position.

Potential function

Generally, if a particle is subjected to both gravitational and elastic forces, the particle's potential energy can be expressed as a potential function

_____

Conservation of energy

When a particle is acted upon by a system of conservative forces only, the total energy, consisting of kinetic and potential energy, of the particle is considered conservative. A mathematical expression showing the conservation of energy of a particle at two instances is given by

_____

Let us consider the case of a ball being dropped to the ground. (Neglecting air resistance)

1. It is initially at rest at a height above the ground. Here its potential energy is at the maximum and the kinetic energy is zero.

2. The ball is falling with a certain speed at midway point. Here its potential energy is half its original value and it has already gained some kinetic energy.

3. The ball is about to hit the ground. Here its potential energy is said to be zero and its kinetic energy is at the maximum.

Conservation of energy of a system of particles

If a system of particles is subjected to conservative forces, then the total energy of the system is also conservative. This is represented by a mathematical expression as

_____