Lecture 4: Structural Analysis

Part 1: Trusses

So far we have only analysed forces and moments on a single rigid body, i.e. bars. Remember that a structure is a formed by ________________________________ and this lecture will investigate forces in such structures.

A ______________ composed of ______________ joined at their ends to form a ________________________ is called a ________________.

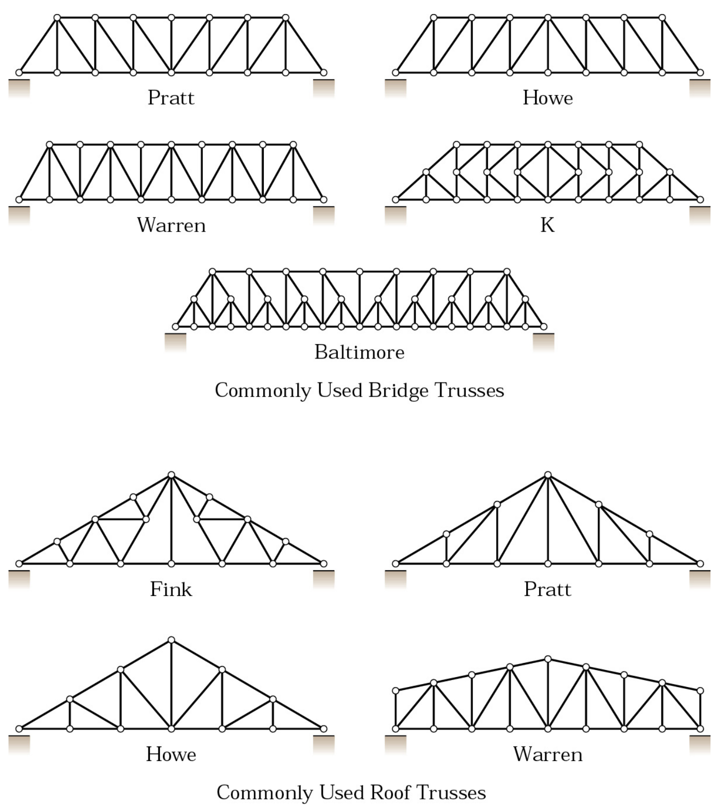


Figure 1 – Examples of different types of trusses

Members of a structure

We will concentrate on structures consisting of ____________________________. These bars are also called members. In a large structure, there may be hundreds or thousands of members and they all carry loads of different magnitudes and directions.

Load carrying members can be classified into three categories

1. Compression

2. Tension

3. Unloaded

Note: There is no specific sign convention for the directions of forces, however it is very important that a consistent system is used throughout the calculation. Here, we will take compression forces to be positive and tension to be negative.

Pin jointed trusses

Recall that a pin joint connects two or more members (bars) together. The joint resists ________________________________ but allows __________________________, i.e. the joint only exerts forces but not moment.

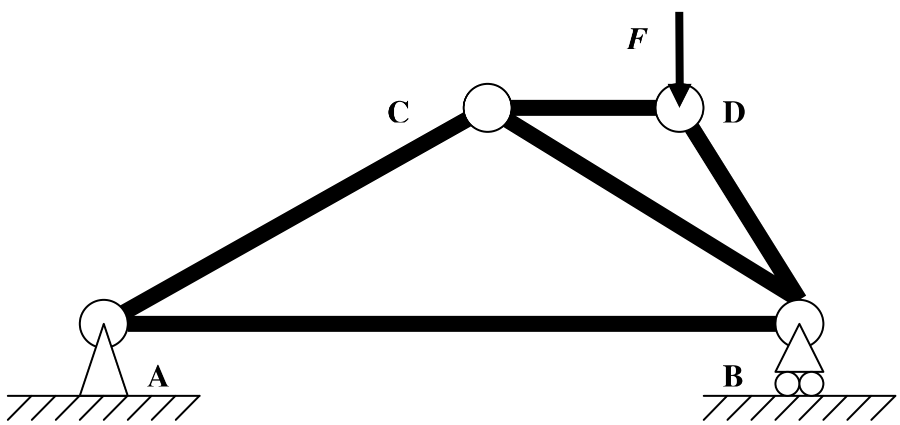


Figure 2

Figure 2 shows an example of a pin jointed truss whose properties are given in the table below. Note that *F* is an externally applied force which acts on the structure at point *C*.

Number of	*Quantity*	*Names*	
Members Reactions Joints			**M =** **R =** **J =**

Table 1

Example of framework analysis

Here, we will attempt to determine all the forces in the structure shown in figure 2.

Step 1

The reaction forces must be determined. There is no externally applied force in the horizontal direction, therefore we can deduce that the reaction $R_1 = 0$. The vertical reaction forces R_2 and R_3 can be given as functions of the externally applied force F.

Step 2

Consider joint A and the members connected to it, namely AB and AC. The joint is in equilibrium and therefore the resultant force at the joint must be zero. By resolving the forces into horizontal and vertical components and using two equilibrium equations in both directions, the two unknown (AB and AC) can be found.

Step 3

At joint C, the vertical equilibrium equation will give the load in the member BC, since the load in AC is already known. The load in member CD is given by the horizontal equilibrium equation.

Step 4

Finally, the load in member BD can be found by applying the equilibrium condition at joint B.

This analysis is based on ____,
therefore this method is also known as the _________________________.

Static determinacy

In the previous part, we were able to determine the loads in ________members of the structure. When such calculation is possible, the structure is said to be ____________ _____________. The condition for a structure to be statically determinate is given by

____

where M represents the number of members

R represents the number of reactions

J represents the number of joints

For a system where______________________________, such a structure is said to be ____. This is because there are more unknowns than available equilibrium equations and one or more members will be indeterminate. Such a system is also called a ____________________ system because it has ____.

A system where ______________________ is called an ______________________ _______________. This is not normally found in standard applications because they are flexible and collapsible under loads.

Examples of redundant frameworks are shown in figure 3.

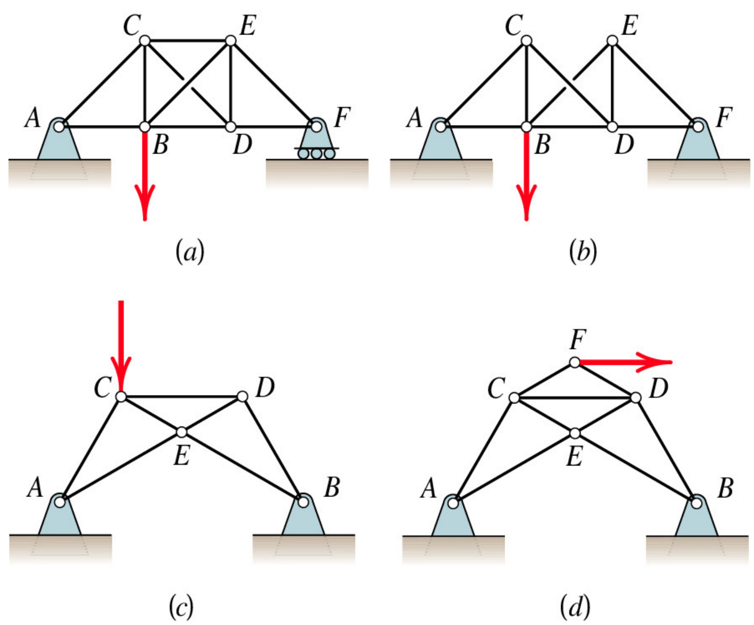


Figure 3

Part 2: Method of Sections

Previously, we resolved the forces in a framework by the ______________________, where the forces around a joint are computed using the equilibrium conditions. An obvious disadvantage of this method is that many calculations will be required if the member that needs to be analysed is far from the reaction forces.

Example 1

Consider the member *FE* in the framework in figure 1. The force in this member can be computed by the method of joints, i.e. ____________________________________

____.

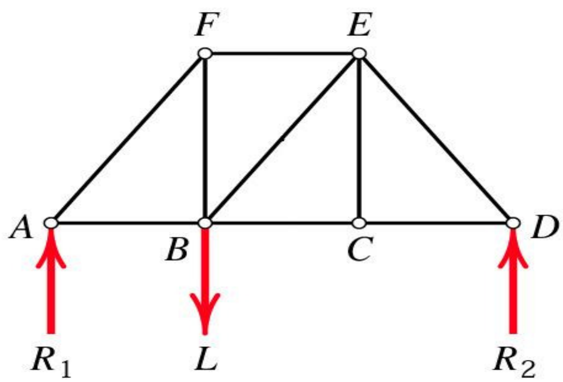


Figure 1

Here, we will introduce the _______________________________. This will help in quickly determining the force in the member in a structure without having to follow the steps shown before. The following describes how the method of sections can be used to determine the force in ____________________ in figure 1.

Step 1

Make an ____.

In this case, the structure is cut vertically through the members *FE*, *BE* and *BC*.

Step 2

Draw _________________________ of the two separate sections with appropriate forces. Each section should be treated as ___________________________________.

The free body diagrams for the two sections are shown in figure 2.

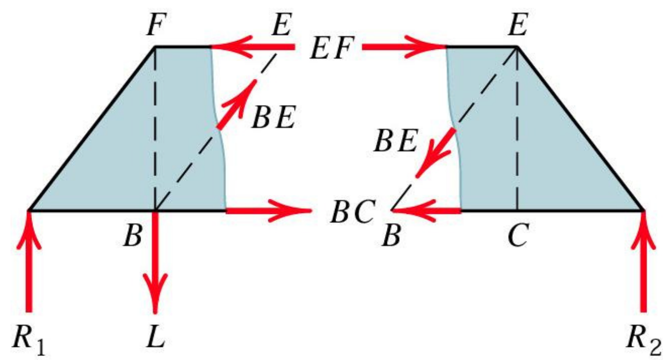


Figure 2

The arrows indicating the direction of the forces in the diagrams show that member *EF* is in tension while members *BE* and *BC* are in compression. This structure is considerably simple and the direction of the forces can be quite easily predicted.

Note that an incorrect direction of the force (tension or compression) in the cut member does not result in an incorrect calculation. It will only lead to the final value of the force being ____________________.

The most important step while making a cut is that the forces in the cut members ___________________________________ at the cut on either side of the sections, i.e. the force *EF* on the left section (towards *F*) must be equal and opposite to the force *EF* on the right section (towards *E*).

Step 3

The required force *EF* can be computed by ___________________________________ _____________________________________. This will eliminate forces *L*, *BE* and *BC* as their moment arms are zero, leaving only forces R_1 and *EF*.

The force *EF* is given by the expression

____

Example 2

Using the framework in figure 1, how would you make a cut to separate the structure into two separate sections in order to determine the force *BC*?

Part 3: Machines

Previously, we looked at frameworks where each member is carrying either compression or tension. Here, we will consider the cases where at least one individual member is a ______________________________. When this is the case, such a structure is called a ___________ or ____________.

Let us define the terms frames and machines as follows

- Frames

  ____

  ____

- Machines

  ____

  ____

Example of an interconnected body with multiforce members

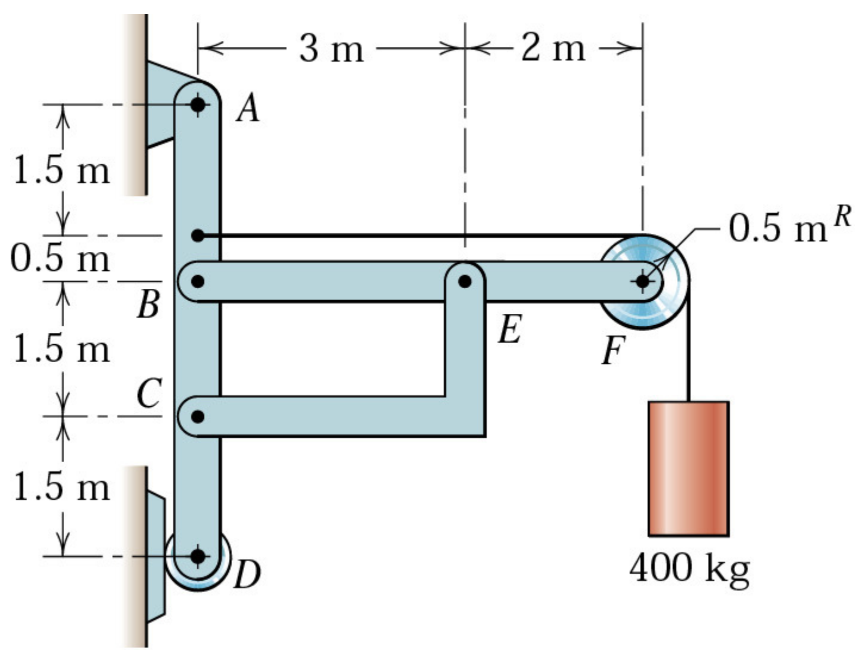


Figure 1

Suppose that we are required to compute the forces acing on each member of the frame. The steps needed to compute the forces are given below.

1. First, always include the ________________________________ and determine the ____________________________________. This is achieved by using the free body diagram of the entire structure. (See figure 2) Note that the forces A_x, A_y, and D are the reaction forces which the supports are acting __________ the structure.

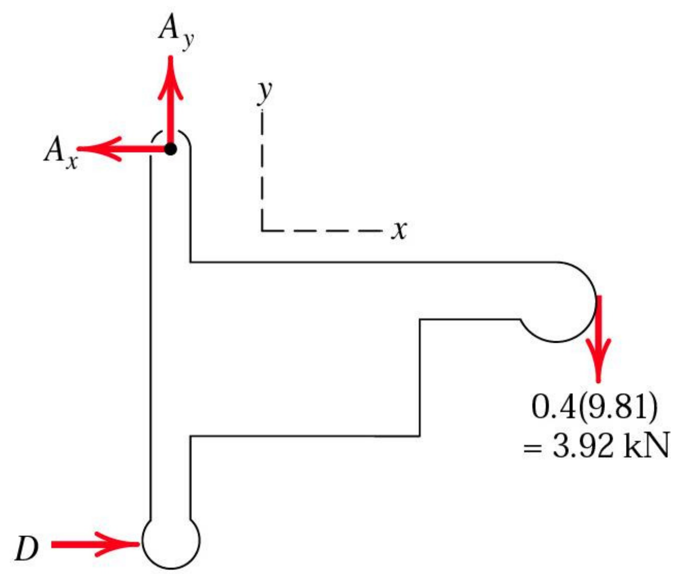


Figure 2.

From the free body diagram in figure 2, we can determine the reaction forces using the equilibrium equations. Hence,

$\sum M_A = 0$	$5.5(0.4)(9.81) - 5D = 0$	$D = 4.32kN$
$\sum F_X = 0$	$A_X - 4.32 = 0$	$A_X = 4.32kN$
$\sum F_Y = 0$	$A_Y - 3.92 = 0$	$A_Y = 3.92kN$

2. Next, we will dismember the frame and consider the equilibrium conditions of each member individually. Each member will have its own free body diagram

Figure 3

Recall the Newton's third law motion which states that the reaction force is always equal and opposite to the action force. In this case, the force acting on connected members must be

____

_____________________________________.

For example, the members AD and BF are pin jointed at point B. Recall that there must be

____ forces acting at a ___________________. These forces are unknown so they are named B_x and B_y, for the horizontal and vertical components, respectively. Each of these two forces acts on the members AD and BF. The magnitude of the force B_x acting on both members must be the same and their directions must be opposite. This also applies to the force B_y.

Note also that the cable can only support ______________________. The 400kg weight produces a 3.92kN cable tension, which is transmitted to the vertical member AD at 0.5m above point B and also to the roller. The cable tension produces horizontal force acting on the member AD whose direction is towards the right hand side, while it also produces a horizontal force towards the left on the roller.

We now have six unknowns in the system, namely B_x, B_y, C_x, C_y, E_x and E_y.

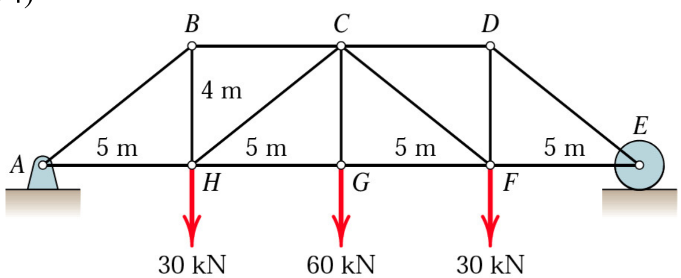
By considering the moment equilibrium of the member CE, we can reduce the number of unknowns using the following relationships.

Finally, all other member forces can be found using the equilibrium conditions on appropriate members. In this case, we will use member BF.

Lecture 4: Exercises

Part 1

(1) Determine the force in each member of the loaded truss. Make use of the symmetry of the truss and of the loading
(Meriam page 174)

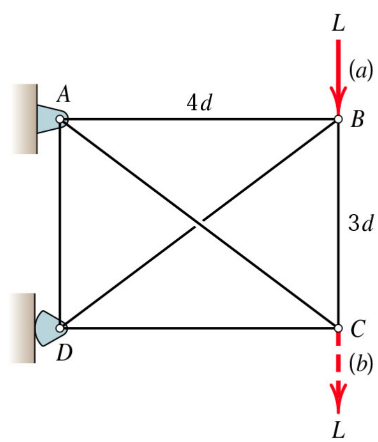


(2) The rectangular frame is composed of four perimeter two-force members and two cables *AC* and *BD* which are incapable of supporting compression. Determine the forces in all members due to the load *L* when

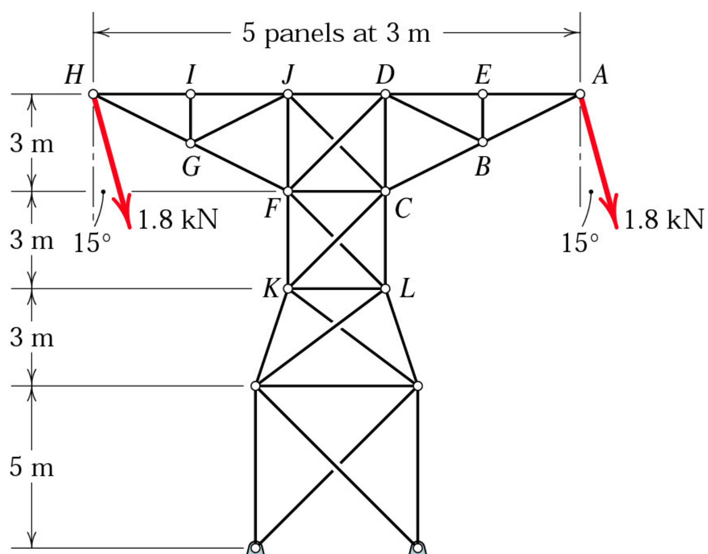
(a) applied vertically at point *B*

(b) applied vertically at point *C*

(Meriam page 176)

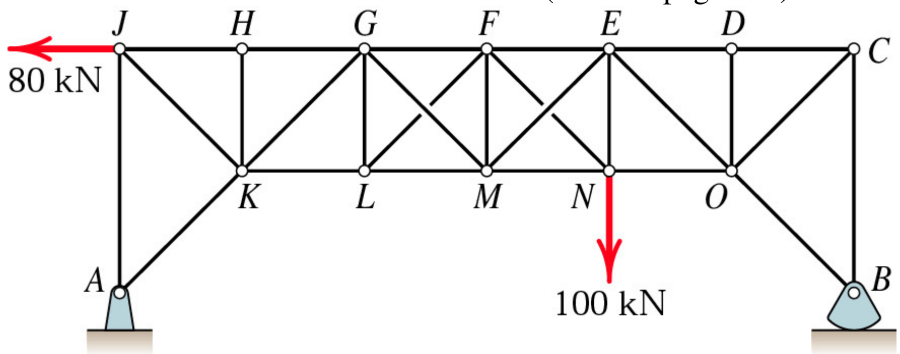


(3) The tower for a transmission line is modeled by the truss shown. The crossed members in the center sections of the truss are cables. For the loads of 1.8kN applied in the vertical plane, compute the forces induced in members AB, DB and CD.
(Meriam page 178)

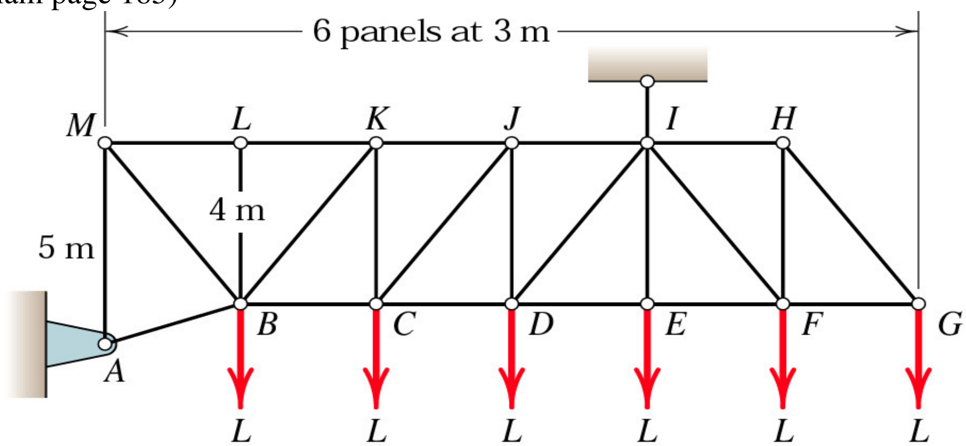


Part 2

(4) Determine the forces in members BC and CG. (Meriam page 184)



(5) Using the method of sections, determine the forces in members *CD, CJ* and *DJ*. (Meriam page 185)



Part 3

(6) The figure shows an eyelet squeezer. An object is placed in the holder to the left of point A. When a pair of forces is applied to the handles, the block at A will slide with negligible friction in a slot. Neglecting the small force exerted by the return spring, determine the compressive force applied to the object when a pair of 80N forces is applied as shown in the figure. (Meriam page 203)