Lecture 2: Force Systems

Part 1: Two-dimensional force systems

Vector quantities can be added or subtracted. Consider the following system of vectors.

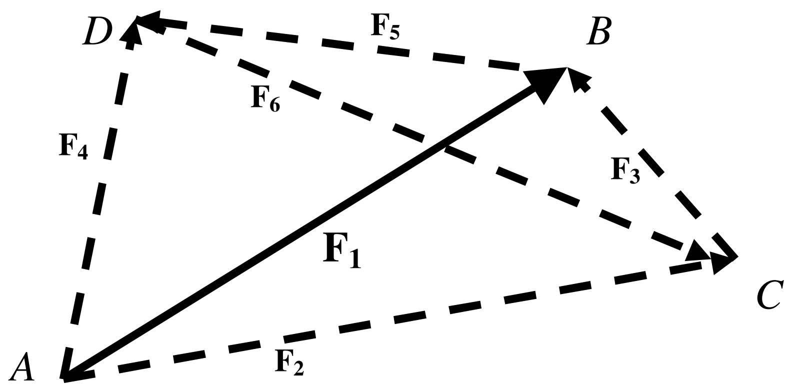


Figure 1

The system shows six vectors namely F_1 to F_6. Vector F_1 connects point A to point B in the direction towards point B. The magnitude of vector F_1 is represented by the _____________ of the vector. The directions and magnitudes of other vectors are illustrated in the figure.

Addition

From figure 1, the vector F_1 begins at point A and is terminated at point B. Now, consider the pair of vectors F_2 and F_3, which also begin at point A and are terminated at point B. This can be mathematically represented as

____

Subtraction

The inverse of a vector is defined as a ____________________________________
____.

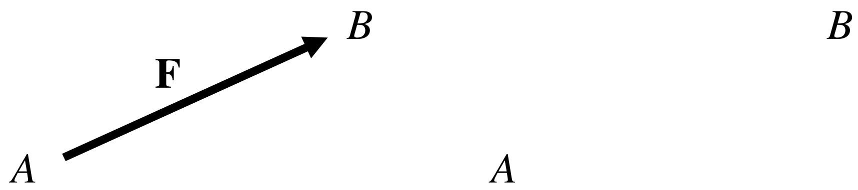


Figure 2

Vectors F_4 and F_5 in figure 1 also connect points A and B, hence the following relationship can be derived.

____

Resolving forces

Forces are _______________________ and can be _____________________________ into several components. The following figure shows that force **F** can be broken down into two components namely the _______________ and the _______________ parts.

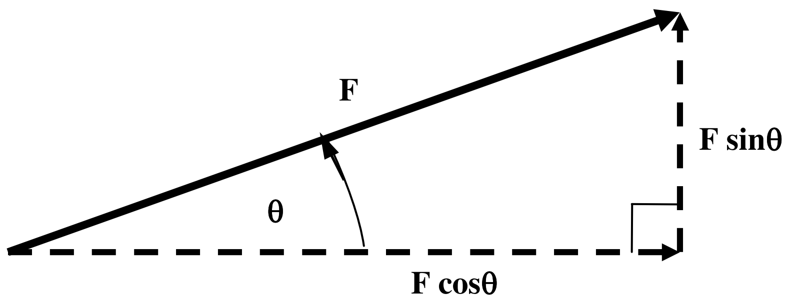


Figure 3

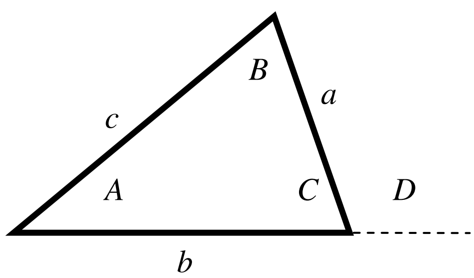
Note that the force F and its components form a right angled triangle. This is very useful because we can use the Pythagorus theorem to determine the magnitude of each component.

For a generic triangle of forces, the sine and cosine rules can be used to determine the magnitude instead.

Sine rule $\dfrac{a}{b} = \dfrac{\sin A}{\sin B}$

Cosine rule $c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = a^2 + b^2 + 2ab\cos D$



Resultant (combined) forces

____

____.

Consider the system of forces in figure 3.

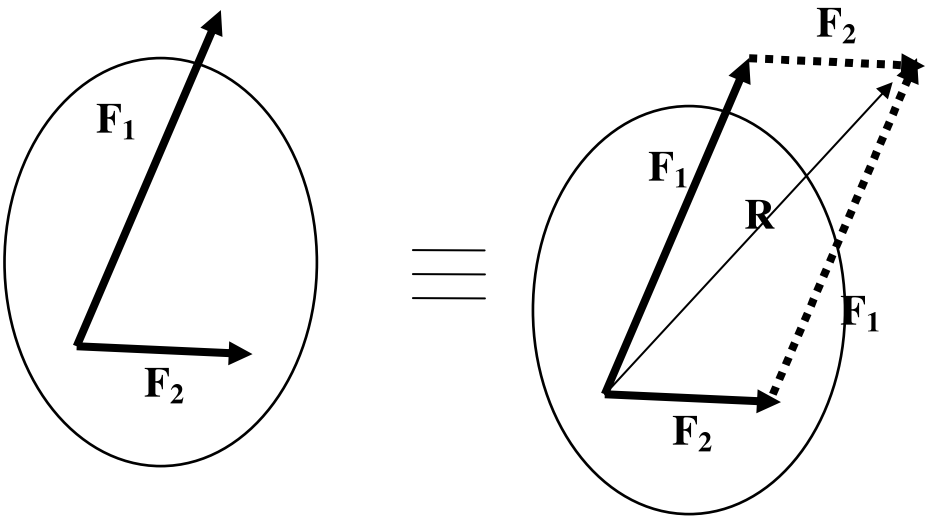


Figure 4

By drawing the system of forces in ______________________________________,
the total effect of forces F_1 and F_2 can be represented by the resultant force R, where

____

Moment

Moment is generated on a body when ______________________________________

____.

The magnitude of the moment is given by

Moment = Force x (perpendicular distance from the rotational axis)

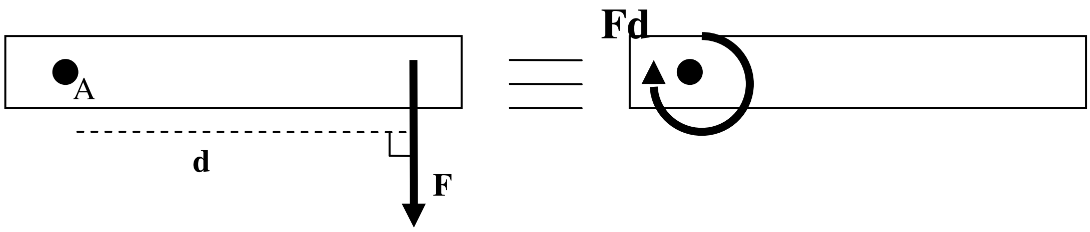


Figure 5

In figure 5, the bar is pivoted at point *A*, i.e. ________________________________.
A vertical force *F* is applied at a horizontal distance *d* from the pivot. This force generates a moment of magnitude *Fd* about point *A*.

Force-couple systems

When a pair of forces of ______________________, _______________________,
and ____
are exerted on a body, the moment they generate is called a __________________.

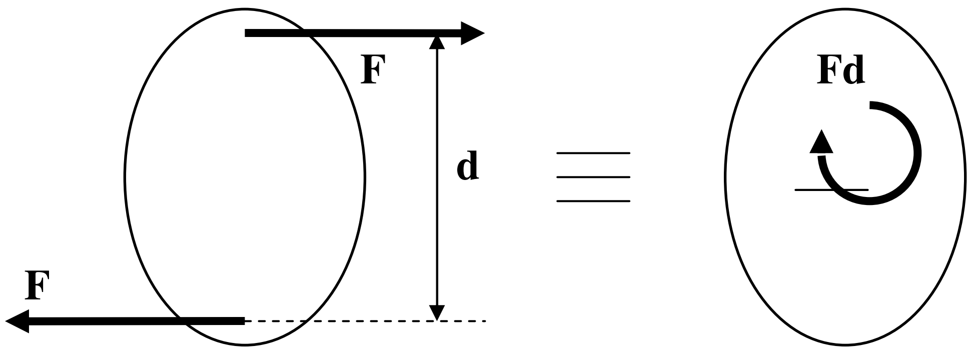


Figure 6

Couples can be combined with forces to produce force-couple systems. Its main application is ____.
In figure 7, a horizontal force F is applied at a distance *d* from point A. This force can be translated onto point A with an additional moment (couple) of magnitude *Fd*.

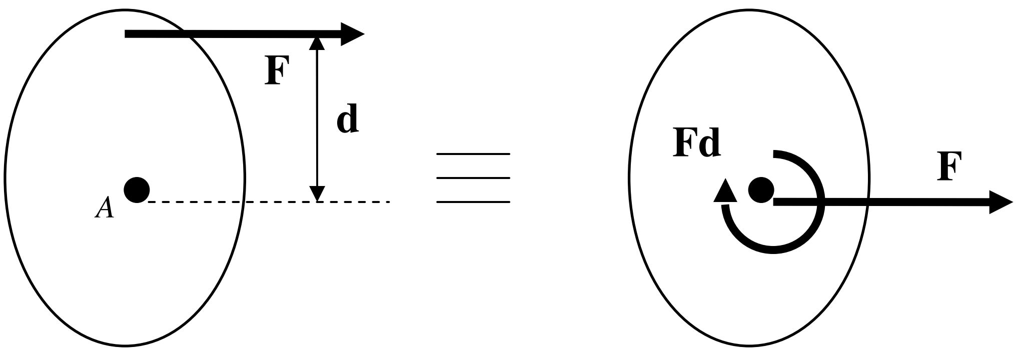


Figure 7

Similarly, a system where only a moment is present can also be represented by a pair of equal, opposite and noncollinear forces.

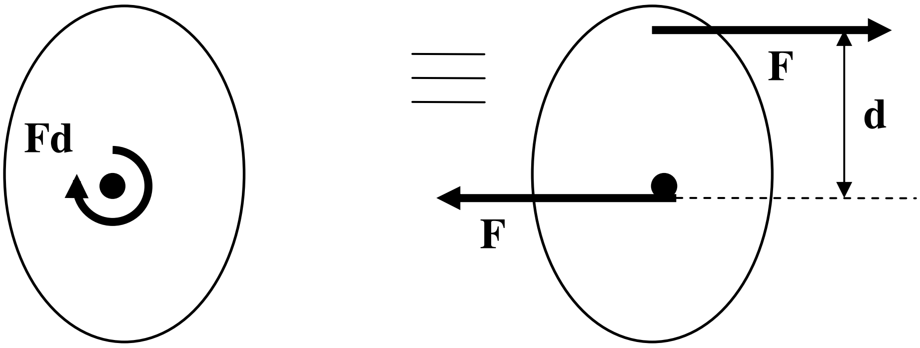


Figure 8

Part 2: Three-dimensional force systems

Systems of forces that appear in general applications are most likely three-dimensional, i.e. at least one force is not acting on the same plane. When this is the case, the __________________________ must be used to analyse such a force system.

Vector system

First, it is very important to establish common background knowledge on the vector system. We will mainly use the ____________________________ here.

The Cartesian coordinates use three directional vectors to define the entire three-dimensional space, i.e. the unit vectors[1] **i, j,** and **k** are directional vectors in the positive direction of x, y and z axes, respectively.

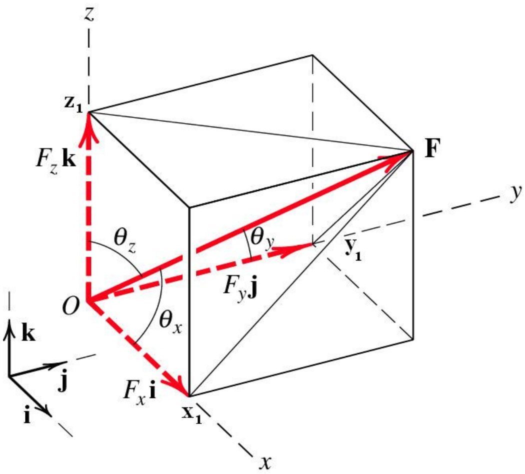


Figure 1

Figure 1 shows a force **F** in a 3-D space. Note that the angles θ_x, θ_y and θ_z are measured on the planes **OFx₁**, **OFx₂** and **OFx₃**, respectively. The properties of this force are given by

Magnitude of x-component of the force	$F_x = F\cos\theta_x$
Magnitude of y-component of the force	$F_y = F\cos\theta_y$
Magnitude of z-component of the force	$F_z = F\cos\theta_z$

[1] A unit vector is a vector whose magnitude is one.

Magnitude of force **F**	$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$
Force **F** defined in vector form	$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$

Dot products of vectors

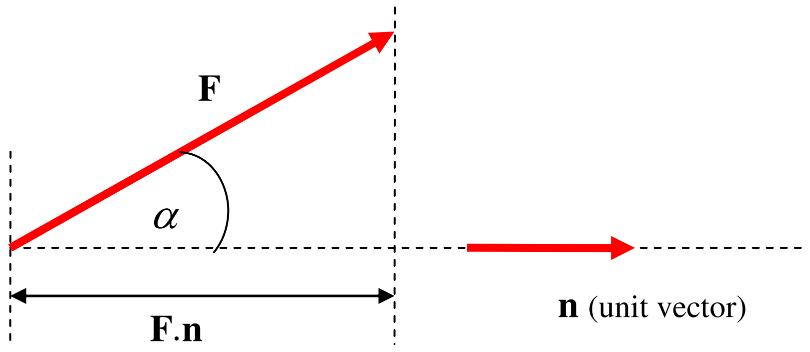


Figure 2

A force is represented by the vector **F**, which makes an angle α to the unit vector **n** as shown in figure 2. The dot product between the two vectors is ____________________ and is given by

$$\mathbf{F} \cdot \mathbf{n} = Fn\cos\alpha \qquad \text{eqn. (1)}$$

The dot product is defined as the ____

____.

In a 3-D space, the angle is measured on the plane which contains both vectors.

Applications of dot products

1. It is used to determine the ____

   ____.

   We can designate the unit vector to point towards any required direction.
2. The previous point can be further extended. As we already have the magnitude of the force in the direction of the unit vector, the force component in that direction may be given by

$$\mathbf{F}_n = (\mathbf{F} \cdot \mathbf{n})\mathbf{n} \qquad \text{eqn. (2)}$$

3. It is used to determine the ____.

   By reversing equation (1), we obtain

$$\alpha = \cos^{-1}\left(\frac{\mathbf{F}\cdot\mathbf{n}}{Fn}\right) \qquad \textit{eqn. (3)}$$

Notes on dot products

Vectors **A** and **B** are in a 3-D space defined by the Cartesian coordinates. The unit vectors **i, j**, and **k** are in the positive directions of *x, y* and *z* axes, respectively. Given that the components of the two vectors are

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \quad \text{and} \quad \mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

Their dot product is given by

____

Cross products of vectors

Using the vectors **A** and **B** from the previous section, their cross product is defined as

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \left(A_yB_z - A_yB_z\right)\mathbf{i} + \left(A_zB_x - A_zB_x\right)\mathbf{j} + \left(A_xB_y - A_xB_y\right)\mathbf{k}$$

Application of cross products

The cross product is used to ____

____ .

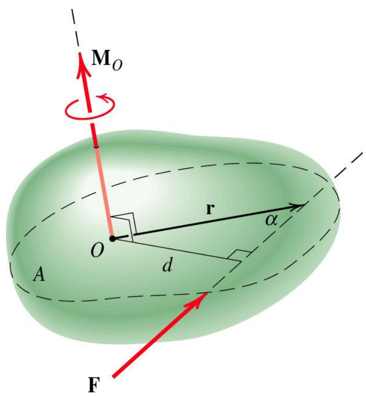


ILLUSTRATION OF RIGHT HAND RULE

Figure 3

The line of action of force **F** in figure 3 is at a perpendicular distance *d* from point *O*, measured on the plane *A*.

The vector **r** connects point O with **any point** which lies on the line of action of force **F.** In this case **r** makes an angle α with the vector **F**.

The moment that force **F** generates about an axis perpendicular to plane A and point O is given by

__________________________ *eqn. (4)*

The direction of the moment is determined by the right hand rule. By pointing the right hand thumb in the direction of the vector $\mathbf{M_0}$, the direction that the other fingers must bend in order to grasp the vector indicates the positive direction of the moment.

Equation (4) can also be used to ______________________ in three dimensions. There must be a pair of parallel and noncollinear forces in space. Vector r is created to join any two points on the lines of action of the pair of forces. See figure 4 for illustration.

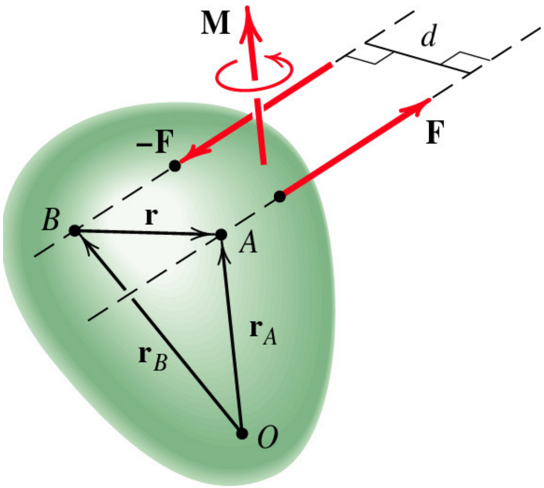


Figure 4

Moments about an arbitrary axis

Suppose that we require to compute the moment about an arbitrary axis, which is not parallel to $\mathbf{M_0}$, i.e. ____.

This is achieved by creating a unit vector **n** in the direction of the required axis $\mathbf{M_\lambda}$. See illustration in figure 5. The moment about $\mathbf{M_\lambda}$ is given by

$$\mathbf{M}_\lambda = (\mathbf{r} \times \mathbf{F} \cdot \mathbf{n})\mathbf{n}$$

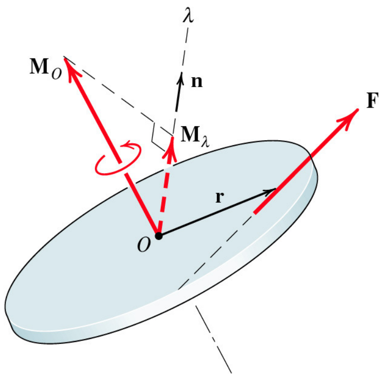
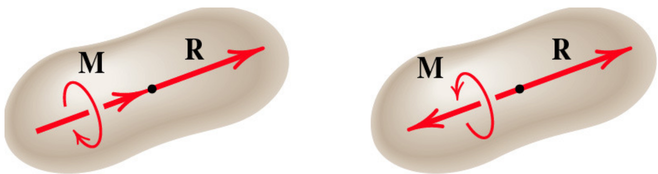


Figure 5

Wrench

When the ____

as shown in figure 6, the resultant is called a _________________.



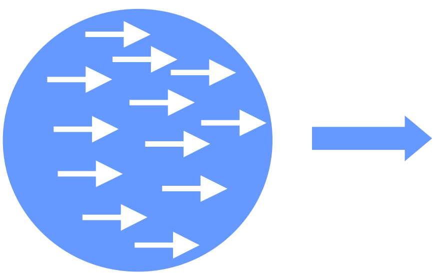
Positive wrench Negative wrench

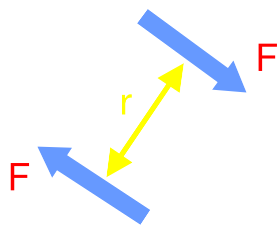
Figure 6

Classification of Vectors

1. **Free Vector**

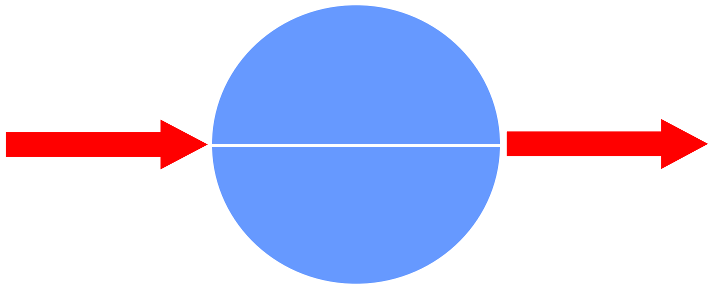
A free vector is one whose action is not confined to or associated with a unique line in space, i.e. displacement of any point in the body with pure translation motion, a couple





2. **Sliding Vector**

A sliding vector has a unique line of action in space but not a unique point of application, i.e. external force acting on a rigid body



3. Fixed vector

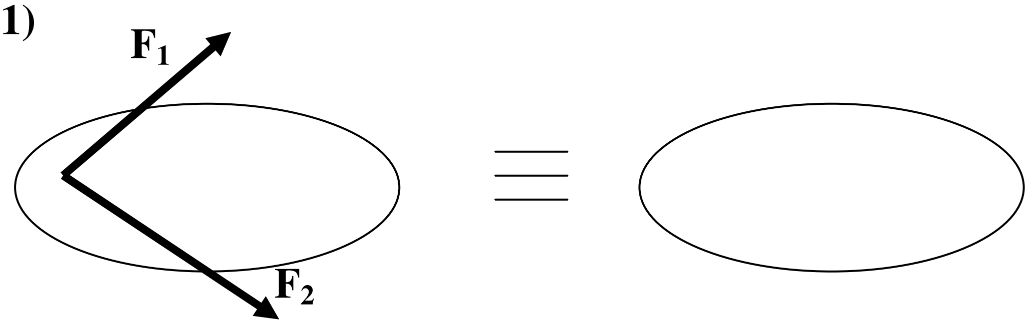
A fixed vector is one for which a unique point of application is specified, i.e. the action of a force on a nonrigid body

Lecture2: Exercises

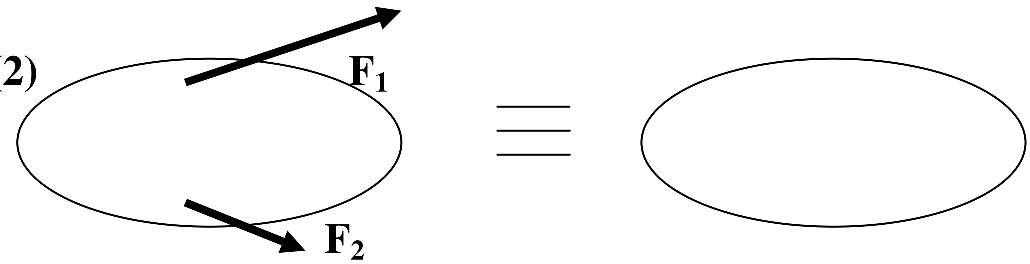
Part 1

Draw on the right figure to graphically determine the resultant force of the following systems. Clearly indicate when the resultant force and moment are nonexistent.

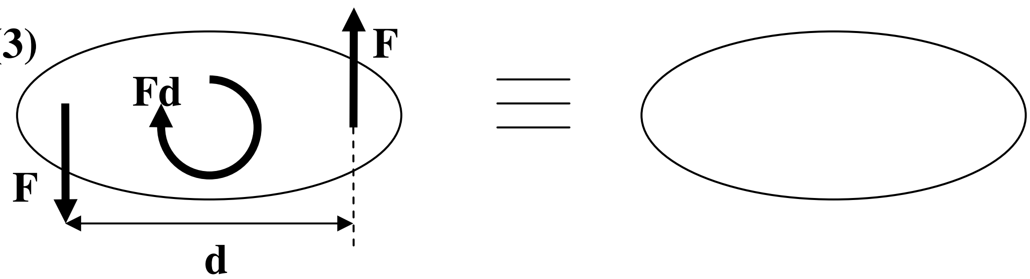
(1)



(2)

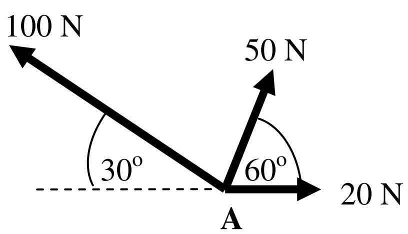


(3)

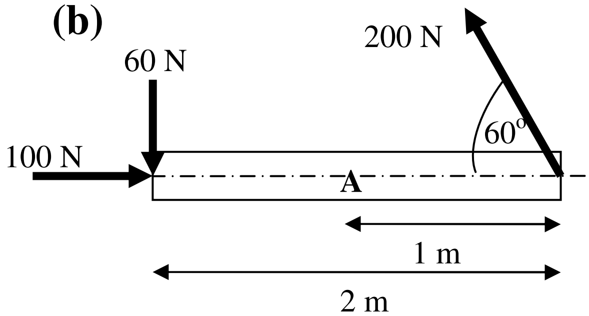


(4) Determine the resultant force and moment at point A. Indicate if the resultant force or moment is zero. Note that forces are not drawn to scale. The bar in (4b) is considered light.

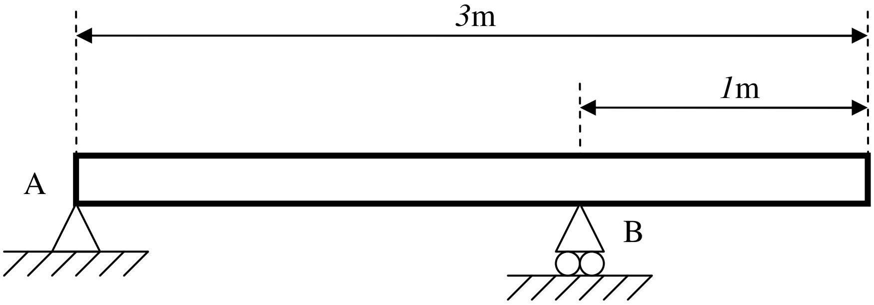
(a)



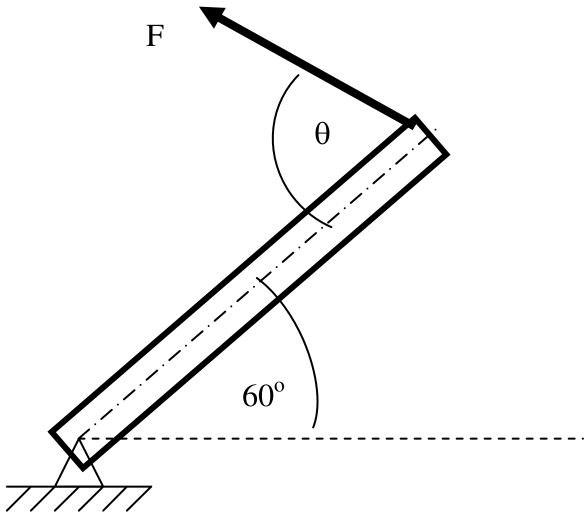
(b)



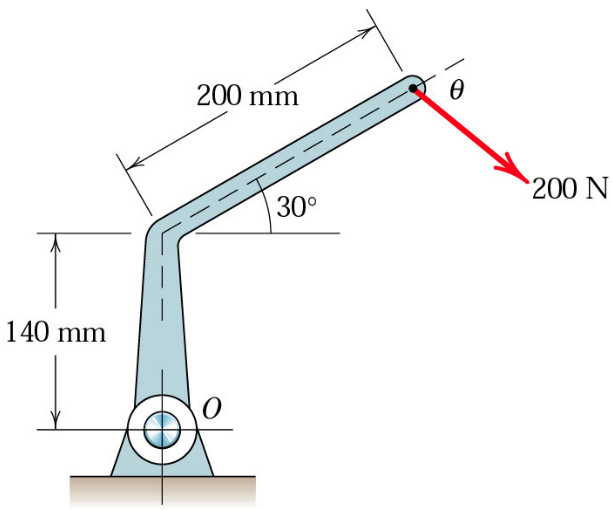
(5) A bar of length *3*m rests in equilibrium on two supports labeled *A* and *B*. The mass of the bar is 20kg. Determine the forces exerted by the two supports.



(6) A bar of mass *M* (kg) and of length *L* (meters) is hinged on one end. A force *F* is applied at an angle θ to the bar on the other end to rotate the bar 60^o from horizontal and keep it in equilibrium. Determine (a) the angle θ and (b) the force exerted by the support.

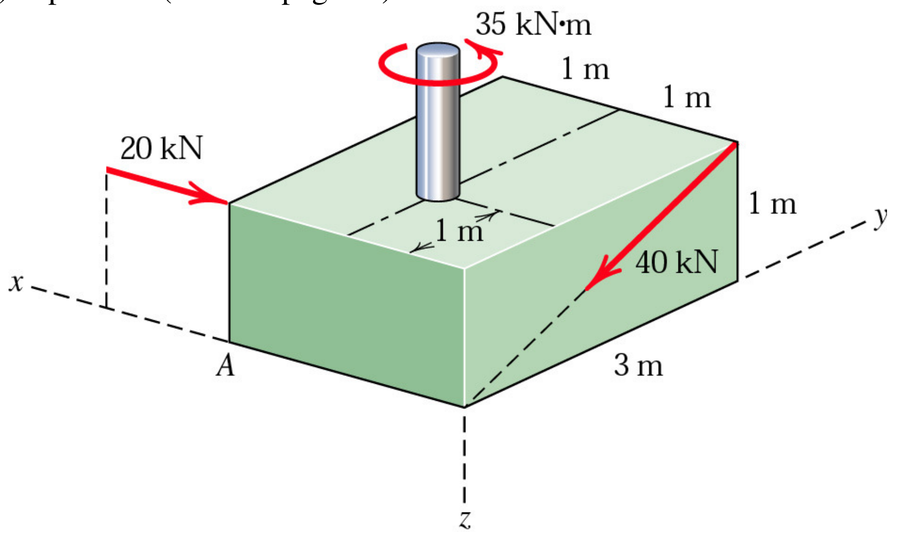


(7) Determine the angle θ which will maximise the moment M_0 of the 200N force about the shaft axis at *O*. Also compute M_0. (Meriam page 45)



Part 2

(1) Replace the two forces and a couple by an equivalent force-couple system (wrench) at point A. (Meriam page 92)



(2) Determine the resultant force acting on the aircraft when one of the engines has failed as shown in the figure. Specify the y- and z-coordinates of the point through which the line of action of the resultant passes. (Meriam page 92)