Lecture 9 – Kinetics of rigid bodies: Impulse and Momentum

Momentum of 2-D Rigid Bodies

Recall that in lecture 5, we discussed the use of momentum of particles. Given that a particle has a ________________, and is travelling with a __________________________, its momentum is given by

____

Now, let us consider a 2-D rigid body of mass m_G translating in a straight line with velocity $\mathbf{v}_G$. We can determine the ____ of the rigid body by summing vectorially the linear momentum of each particle that makes up this body, i.e.

_________________________ [Eqn.1]

The term **L** in equation 1 denotes the rigid body linear momentum.

When a 2-D rigid body undergoes a rotational motion, its ___________________________ is given by

_________________________ [Eqn.2]

where H_O is the angular momentum of the rigid body about point O

I_O is the moment of inertial computed at point O

ω Is the angular velocity of the rigid body

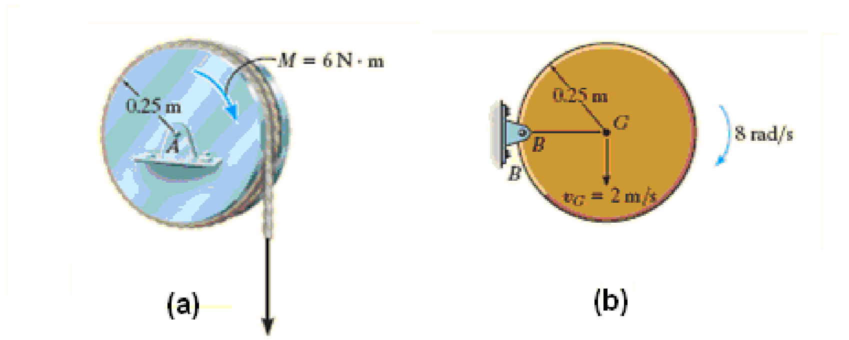


Figure 1

The position of point *O* is arbitrary and should be selected appropriately for specific problems. For example in figure 1(a), the disc rotates about its ____________________ and the moment of inertia is given by ____________________, Whereas in figure 1(b), the disc is rotating about the pin support at point B which is on the circumference, hence the moment of inertia in this case is ______________________.

Note that we can ___________________ the axis of rotation from *A* to *B* (for the disc in figure1) and the values of the moment of inertia by using the relationship

____

We will encounter ________ types of motion by a 2-D rigid body, namely ___________ ________________, _________________________, and ______________________.

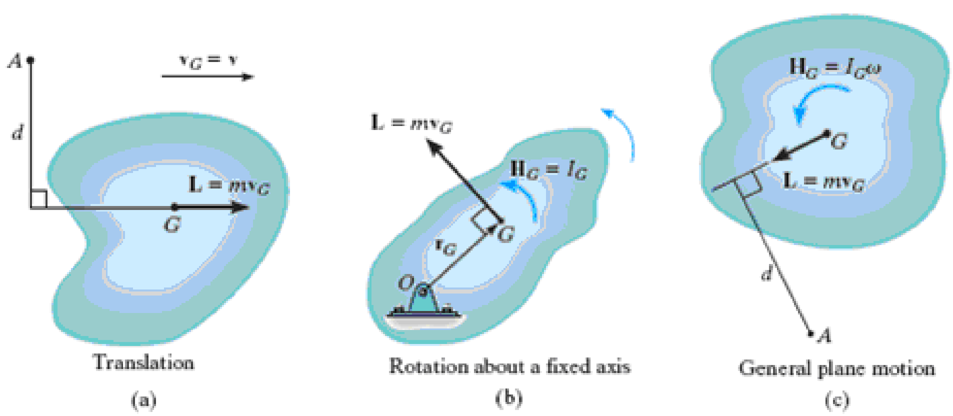


Figure 2

Note that we will usually compute the momentum at the centre of mass of the object. This point is denoted by the subscript *G*.

Pure Translation

For a 2-D rigid body undergoing only translational motion, its momentum is given by

____

____

Rotation About a Fixed Axis

For a 2-D rigid body undergoing rotational motion about a fixed axis, its momentum is given by

____

____

or we can also compute the angular momentum at the rotational axis, denoted by point O in this case

____

General Plane Motion

For a 2-D rigid body in a general plane motion, i.e. it is undergoing both translational and rotational motions, its momentum is given by

____

____

Principle of Impulse and Momentum

Linear Impulse and Momentum

Recall that a linear impulse for a particle is given by______________________________. It is a vector quantity which quantifies the effects of a force during the time the force acts. It has the same direction as the force, and its magnitude has units of *N sec*.

The principle of linear impulse and momentum states that the _______________________ acted on the body by external forces during the time interval t_1 to t_2 is equal to ________ ____ of the body during the same time interval. Rewriting this in a mathematical expression yields

____

____ [Eqn.3]

Angular Impulse and Momentum

The principle of angular impulse momentum of a 2-D rigid body takes a similar form as the linear momentum. Hence, for a body undergoing general plane motion, this is given by

____

____ [Eqn.4]

Combination of Linear and Angular Impulse and Momentum

We can summarise the principle of impulse and momentum for a rigid body with planar motion, i.e. translation on *x-y* plane and rotation about *z* axis, as

____

____

____

Conservation of Momentum

Linear Momentum

From equation 3, we see that if the ______________________________ acting on the body over the time interval _________________, the linear momentum of the system must be ____, i.e.

$$0 = [\text{Final linear momentum}] - [\text{Initial linear momentum}]$$
$$[\text{Final linear momentum}] = [\text{Initial linear momentum}]$$

____

We can apply the concept of momentum conservation when the linear impulses are ____. Examples of these circumstances are small forces acting over very short period of time.

Angular Momentum

$$0 = [\text{Final angular momentum}] - [\text{Initial angular momentum}]$$
$$[\text{Final angular momentum}] = [\text{Initial angular momentum}]$$

____

The conservation of angular momentum takes a similar form as the linear part.

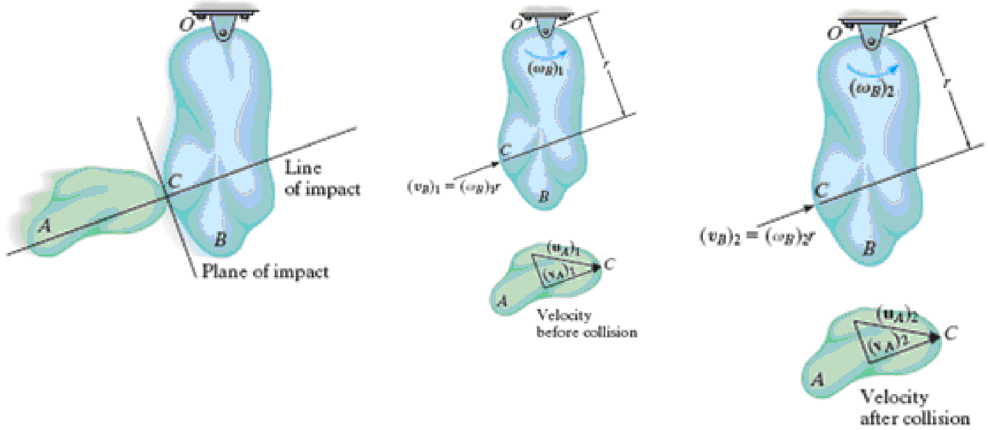
An example of conservation of angular momentum in practice is when a diver athlete executes a somersault. He tucks in his limps close to his body in order to reduce his body's moment of inertia, so that his angular velocity (in this case, spin rate) increases.

Eccentric Impact

An eccentric impact is an impact where the mass centres of two rigid bodies __________

____

(See figure 3a). In the case where alignment is present, we use the particle impact analysis as discussed in lecture 4.



(a) (b) (c)

Figure 3

Recall the ____ that we encountered before while analysing central impact. The coefficient of restitution is defined as the ratio of ____ of the points of contact just after impact to the ____ of these points.

Therefore, for the eccentric impact, the coefficient of restitution is given by

____

where v_A denotes the velocity of body A in the direction of the line of impact

v_B denotes the velocity of body B in the direction of the line of impact

subscript 1 denotes the values before impact

subscript 2 denotes the values after impact