Lecture 8 – Kinetics of a rigid body: Work and Energy

Work of a Force

A force is said to have done work when ___________________________________

____.

Work of a Variable Force

A variable externally applied force **F** moves the object along the path *s*. If the vector **r** represents the displacement of the object, the work of a variable force **F** along path s is given by

____________________________________ (EQN. 1)

Work of a Constant Force

The magnitude and direction of a constant force must not change over time. If an object is acted upon by a constant force and undergoes a translation s, equation 1 simplifies to

__________________________________ (EQN. 2)

The term in the brackets represents the force component in the ____________________.

Work of a Weight

The weight of an object is said to do work (or negative work) when the object undergoes a vertical displacement. The measurement of vertical displacement is usually defined as positive upwards on the *y*-axis, hence the expression for work of a weight is given by

____

Work of a Linear Spring Force

A linear spring generates a linearly increasing force with elongation or compression of the spring original length. Note that when stretched or compressed, the spring always exerts the force which tries to ____.
Hence, the direction of displacement of the body attached to the free end of the spring as

shown in the figure is ____________________________________ to the direction of the force. The resulting work done by a linear spring force is given by

____

where ______________________

Work of a Couple

A couple is said to have done work when the object on which the couple is applied undergoes ____ about an axis perpendicular to the plane of motion. It is given by the expression

____

For a special case where the moment *M* has a __________________________________, the above expression simplifies to

____

Kinetic Energy

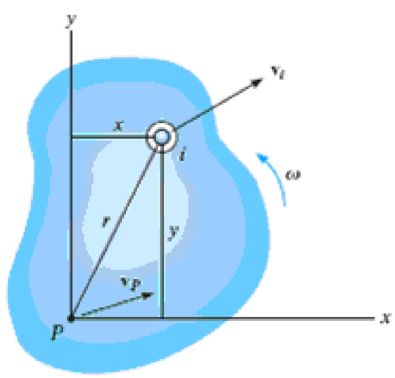

Figure 1

Let us consider an arbitrarily shaped two-dimensional rigid body as shown in figure 1. It is rotating with an angular velocity ω *rad/sec*. Both points *P* and *i* are arbitrarily chosen. The kinetic energy of element *i* is given by

____

The total kinetic energy of this 2D body is therefore by the sum of kinetic energy of all elements, i.e.

____________________________________ (EQN. 3)

From kinematics of a 2D rigid body, the velocity of element i is given by

$$
\begin{aligned}
\mathbf{v}_i &= \mathbf{v}_P + \mathbf{v}_{Pi} \\
&= (v_P)_x \mathbf{i} + (v_P)_y \mathbf{j} + [\omega \mathbf{k} \times (x\mathbf{i} + y\mathbf{j})] \\
&= ((v_P)_x - \omega y)\mathbf{i} + ((v_P)_y + \omega x)\mathbf{j}
\end{aligned}
$$

Hence, the square of magnitude of velocity of element i is

$$
\begin{aligned}
(v_i)^2 &= ((v_P)_x - \omega y)^2 + ((v_P)_y + \omega x)^2 \\
&= (v_P)^2 - 2(v_P)_x \omega y + 2(v_P)_y \omega x + \omega^2 r^2
\end{aligned}
$$

Substituting the expression for the magnitude of velocity into equation 3 yields

$$
T = \frac{1}{2}\left(\int_m dm\right)(v_P)^2 - (v_P)_x \omega \left(\int_m y\ dm\right) + (v_P)_y \omega \left(\int_m x\ dm\right) + \frac{1}{2}\omega^2 \left(\int_m r^2 dm\right)
$$

____ (EQN.4)

where $\bar{x}$ is the distance between point P and the centre of mass in the x-direction

$\bar{y}$ is the distance between point P and the centre of mass in the y-direction

I_P is the body's moment of inertia about the z-axis passing through point P

Since point P has been arbitrarily chosen, we can freely change it to a convenient point in order to simplify the expression for the total kinetic energy. If point P coincides with ____ then equation 4 becomes

_______________________________ (EQN. 5)

This equation represents a general form of the kinetic energy of a rigid body under a general plane motion. It is a sum of the body's ________________________________ ____ about the mass centre.

Equation 5 can be simplified when applied to systems under certain constraints. For example, if only translation motions are allowed, the expression for total kinetic energy becomes

When the body undergoes a pure rotational motion about a fixed axis at point O, its kinetic energy is given by

where I_O is the moment of inertia of the body computed about an axis perpendicular to the plane of motion through point O. It is usually calculated using the parallel axis theorem.

Gravitational Potential Energy

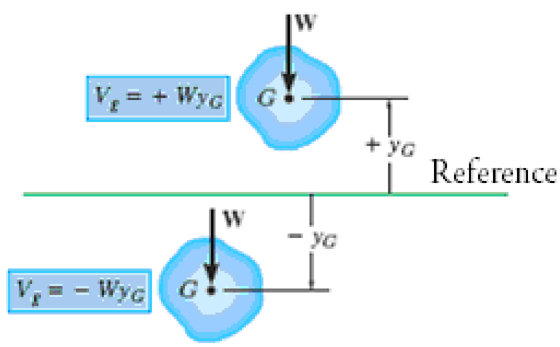

Figure 2

It is usually acceptable to assume that the total weight of a body is concentrated at its centre of gravity, and its gravitational potential energy, relative to a fixed reference height, is given by

where $W = mg$ is the weight of the body

y_G is the vertical distance between the centre of mass and the reference height

Elastic Potential Energy

The elastic potential energy stored in an ____ spring is given by

____

where k is the spring stiffness

s is the elongated or compressed length of the spring

Principle of Work and Energy

A similar expression as one used in the kinetic of a particle will be applied here. The sum of body's initial energy (kinetic and potential), plus the work done by all external forces and couples acting on the body as it moves from its initial to final position, is equal to the sum of its energies at the final position.

_____________________________________ (EQN. 6)

Note that the gravitational potential energy terms in equation 6 are often omitted because the change $\Delta V = V_1 - V_2$ is the _________________________ by the weight of the object. This term can be included in the sum of work done ΣU_{1-2}. Therefore equation 6 can be rewritten as

____

Conservation of Energy

Given that an object is displaced from an initial position to a final position by means of _________________________________ forces only, i.e. nonconservative forces such as friction is not present in the system, $\Sigma\left(U_{1-2}\right)_{\text{nonconservative}} = 0$. We can rewrite equation 6 as

____

This equation represents the conservation of mechanical energy. Examples of mechanical systems which we can use this relationship are smooth, pin-connected rigid bodies, bodies connected by inextensible cords, and bodies in mesh with other bodies.