Lecture 6: Planar Kinematics of Rigid Bodies: Part 2

Absolute Motion Analysis

When motion of a mechanical system is analysed by a stationary observer, it is said to undergo an ____ analysis. In another word, we will use a ________________________ frame of reference when we perform an absolute motion analysis.

Remember that a rigid body general plane motion consists of, possibly simultaneous, ____ motion.

Figure 1

An example of a dump truck is shown in the above figure. The observer is positioned several metres from the back of the truck and sees that the bin is being tilted by means of a hydraulic shaft. Relative to the observer, points A and B are fixed, whereas the position of point *C* is not. Using the cosine rule the length of the hydraulic shaft is given by

____

We can then determine the speed at which the hydraulic shaft extends and the angular velocity of the bin (about point *C*) by ____ the above expression with respect to time. We then obtain

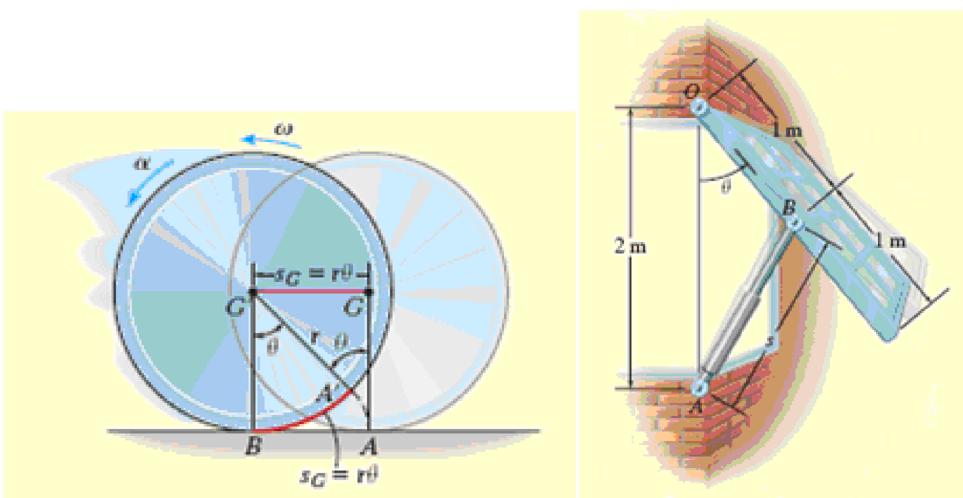
____

Note: Be careful when performing the differentiation with respect to time because every variable must be differentiated independently. The ________________________ must be used when working with a product of more than one variable.

Examples

(a) Determine the velocity and acceleration of point G if the disc rolls without slipping

(b) Determine the angular velocity and acceleration of the window about the hinge at the instance $\theta = 30^o$, if the hydraulic cylinder extends at a constant rate of v_s *m/s*.



(a) (b)

Figure 2

Relative Motion Analysis

Let us consider an arbitrarily shaped object undergoing a general plane motion as shown in figure 3. The position of point A on the body can be described using a vector relative to the origin, hence it is described using a fixed frame of reference. However, the position of point B is then measured relative to point A and we will use a vector which is now in the x` and $y$`- axes instead.

The x\` and $y$\`-axes form a translating reference, thus the study of motion of point B is a ___________________ analysis. Note that the x\` and $y$\`-axes are _________________ to the original x- and y-axes, respectively

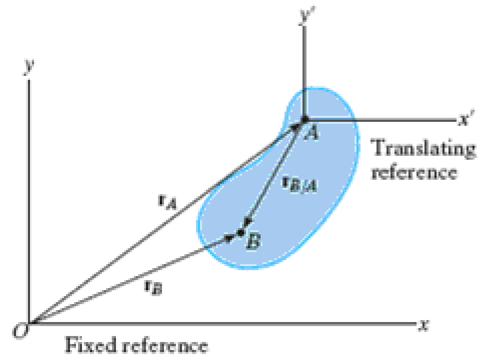


Figure 3

The position of point B is given by the vector

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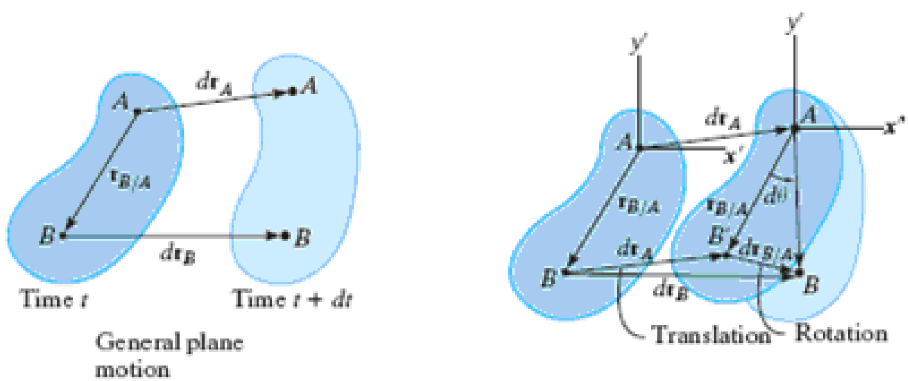
where the vector $\mathbf{r}_{AB}$ locates point B with respect to point A

Relative Motion Displacement



(a) (b)

Figure 4

Suppose that a rigid body undergoes a displacement $d\mathbf{r}$ during an instance dt. The displacements of points A and B are denoted by $d\mathbf{r}_A$ and $d\mathbf{r}_B$, respectively. The relationship of these two displacements are given by

______________________________ (EQN. 1)

where the vector $d\mathbf{r}_{AB}$ represents the ______________________________ of point B relative to point A.

Relative Motion Velocity

We can directly differentiate equation 1 with respect to time to obtain the equation for velocity. The expression now becomes

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where	$\mathbf{v}_A$	represents the velocity of point A (fixed frame of reference)
	$\mathbf{v}_B$	represents the velocity of point B (fixed frame of reference)
	$\mathbf{v}_{AB}$	represents the relative velocity of point B *relative* to point A

The distance between points A and B always ______________________________ since both points are located within a rigid body, hence on the translating reference, point B can only rotate about point A. The relative velocity can be given by the vector

$$\mathbf{v}_{AB} = \boldsymbol{\omega} \times \mathbf{r}_{AB}$$

where $\boldsymbol{\omega}$ is the angular velocity of the body measured from a fixed reference. Therefore, the velocity equation can be rewritten as

______________________________ (EQN. 2)

Relative Motion Acceleration

The acceleration equation of two points on a rigid body can be obtained by differentiating equation 2 with respect to time to give

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where $\mathbf{a}_A$ denotes the acceleration of point A

$\mathbf{a}_B$ denotes the acceleration of point B

$\boldsymbol{\alpha}$ denotes the angular acceleration of the body

$\boldsymbol{\omega}$ denotes the angular velocity of the body

$(\mathbf{a}_{AB})_t$ denotes the relative tangential acceleration of B with respect to A

$(\mathbf{a}_{AB})_n$ denotes the relative normal acceleration of B with respect to A

$\mathbf{r}_{AB}$ denotes the relative position vector from A to B

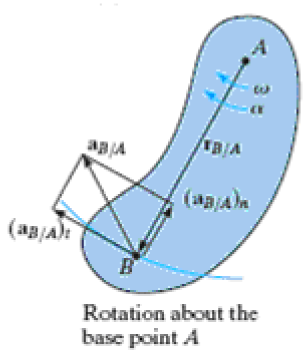

Figure 5

<u>Example</u> Determine the velocity of collar A at the instance shown in figure 6.

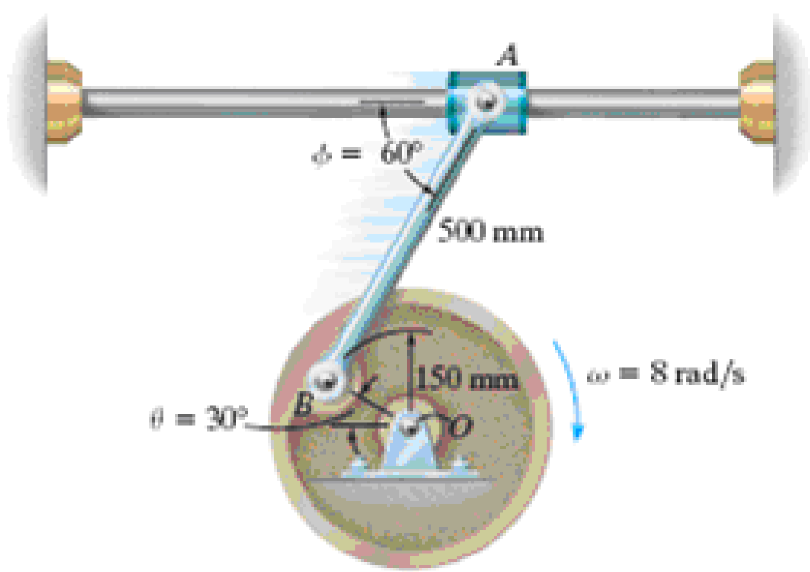

Figure 6

Instantaneous Centre of Zero Velocity

Let us consider the motion of a bicycle wheel as shown in figure 7. The wheel is rotating ____ with an angular velocity ω. At this instant, the velocity of point *IC* (which is a point on the wheel) is __________.

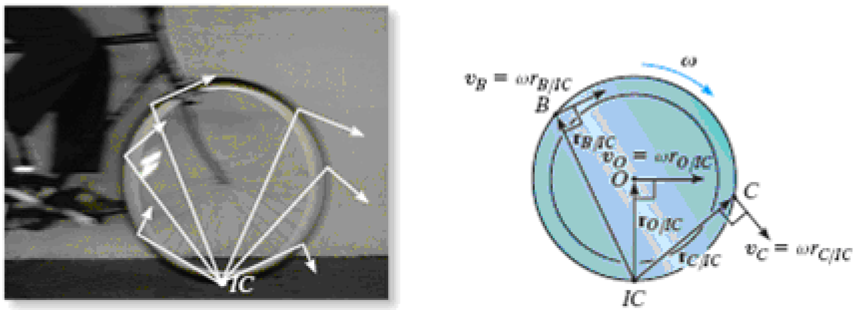

Figure 7

Using the instantaneous zero velocity condition at point IC, the velocities of other points on the wheel can be directly computed using the relationship

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The location of the instantaneous centre of zero velocity (called IC for short) can be determined using one of the four methods shown in figure 8.

(a) velocity of point A $\mathbf{v}_A$ and the angular velocity of the body $\boldsymbol{\omega}$ are known

(b) velocities of points A and B, $\mathbf{v}_A$ and $\mathbf{v}_B$, are not parallel

(c) velocities of points A and B, $\mathbf{v}_A$ and $\mathbf{v}_B$, are opposite and parallel (but do not coincide)

(d) velocities of points A and B, $\mathbf{v}_A$ and $\mathbf{v}_B$, are parallel and in the same direction but not equal in magnitude

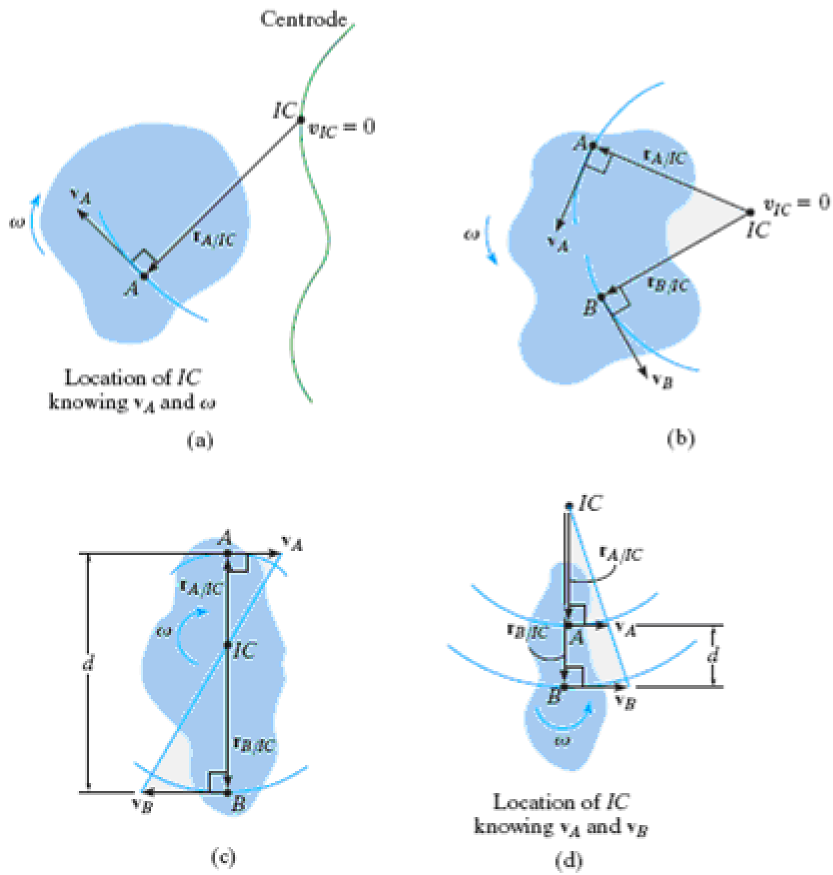

Figure 8

Example – Solve the problem in figure 6 using the instantaneous zero velocity method