Lecture 5 – Kinematics of a 2-D rigid body (Part 1)

In previous lectures we studied motion and effects of forces on ____________________ in two- and three-dimensional spaces. From this point forward, we will perform similar analyses again but on ______________________________________ instead of particles.

The main difference which we will experience is that the _________________________ of a rigid body. A general plane motion of a rigid body, i.e. rigid body motions on a 2-D plane, consists of ____.

Essentially, a rigid body has a total of six degree-of-freedom in a 3-D space which are translations and rotations in the x-, y- and z-directions.

Vector representation of translation motion

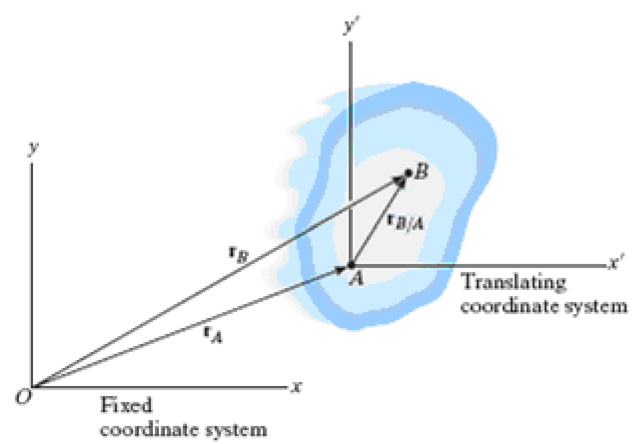

Figure 1

Consider a rigid body which is free to translate on the x-y plane. Points A and B are located on the body. The position vectors of these two points are given by

____________________________________ (EQN. 1)

where $\mathbf{r}_A$ represents the positional vector of point A relative to a reference

$\mathbf{r}_B$ represents the positional vector of point B relative to a reference

$\mathbf{r}_{AB}$ represents the vector which connects point A to point B

The vector form of the velocity of the rigid body can be obtained by differentiating the expression in equation 1 with respect to time. This is given by

__________________________________ (EQN. 2)

The term $\frac{d\mathbf{r}_{AB}}{dt} = 0$, since the magnitude of $\mathbf{r}_{AB}$ cannot change over time.

Finally, the acceleration vector of the rigid body undergoing translation motions is given by

__________________________________ (EQN. 3)

Vector representation of rotation about a fixed axis

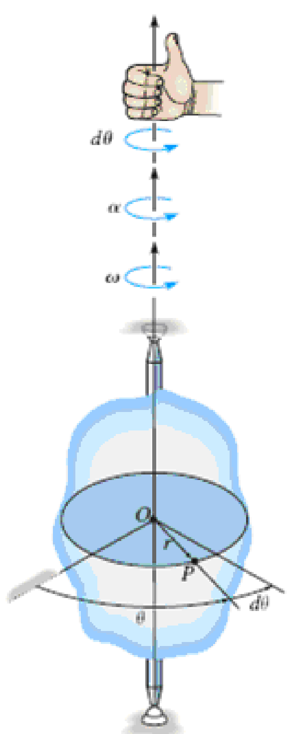

Figure 2

We will consider the disc element rotating about the vertical axis which is part of a rigid body the figure. First, it is important to establish the positive direction of rotation. We will use the ____. The thumb points toward the positive direction of the rotation axis and the direction which the fingers grasp around the axis indicate the positive direction for rotation.

The angular motion will be denoted by the variable θ. The unit for angular motion is usually radians. Measurements of angle θ must always be performed with respect to a fixed reference.

The angular velocity is the time rate of change of the angular motion, hence its mathematical representation is given by

____

Finally, the angular acceleration is given by

____

The units for angular velocity and acceleration are usually *rad./sec*. and *rad./sec.*^{2}, respectively.

For a constant angular acceleration $\alpha = \alpha_c$, we can derive the following

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____

Motion of a rotating point

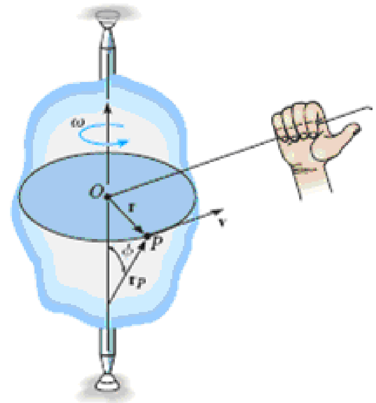

Figure 3

Consider point P on the rotating disc. Its position is simply given by the vector **r**, which connects point O to point P.

The velocity at point P is tangential to the circle, so it is given by

____

Note that the positive direction of a cross product can be determined using the right hand rule.

Finally, the acceleration of point P is given by

____

The subscripts *t* and *n* denote ____ directions, respectively. Hence, the tangential acceleration is simply a time derivative of the velocity. The normal acceleration always exists for angular motion and its direction must be _________________________ of rotation, which causes its value to be negative.

Note that the scalar (magnitude) quantities for acceleration can also be given by

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