Lecture 4 – Kinetics of a particle Part 3: Impulse and Momentum

Linear impulse and momentum

Starting from the equation of motion for a particle of mass *m* which is subjected to an arbitrary force $\Sigma\mathbf{F}$

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where **a** and **v** are the particle's ______________________ and _____________________, respectively. Both are measured in an __________________________ frame of reference.

Rearranging the terms and integrate the equation of motion using the limits:

- $\mathbf{v} = \mathbf{v_1}$ at $t = t_1$
- $\mathbf{v} = \mathbf{v_2}$ at $t = t_2$

to obtain the following equations

$$\Sigma\int_{t_1}^{t_2} \mathbf{F}\ dt = m\int_{\mathbf{v}_1}^{\mathbf{v}_2} d\mathbf{v}$$

__________________________________ (EQN. 1)

The term ________________ is referred to as *the particle's* __________________________. The vector **L** has the same direction as the velocity of the particle and its magnitude is *mv*. Its unit is given by ___________________________. Recall Newton's second law of motion which states that the force acting on a body is equal to the rate of change of its momentum.

The term __________________________ is referred to as *the* ________________________. It is a vector quantity which quantifies the effects of a force during the time the force acts. It has the same direction as the force, and its magnitude has units of _____________.

Graphically, the impulse is determined by the measuring the area under the force-time graph, between specific limits (See figure 1)

Equation 1 also represents the principle of linear impulse and momentum. Rearranging equation 1 to obtain

____ (EQN. 2)

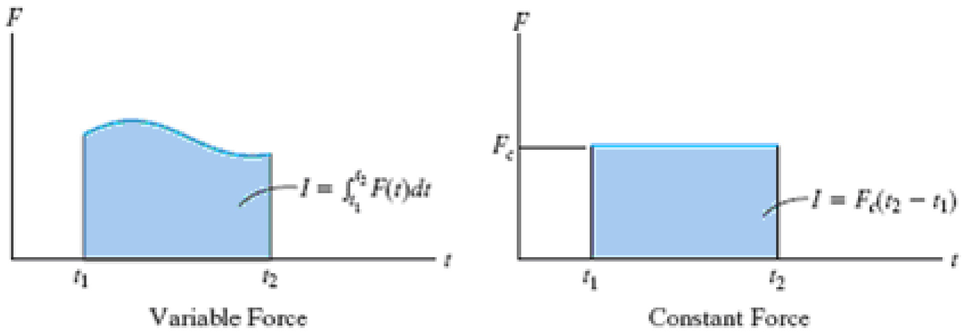

Figure 1

Equation 2 states that the initial momentum of the particle at time t_1 plus the sum of all impulses applied to the particle between time t_1 and t_2 must be equivalent to the final momentum of the particle at time t_2.

Equation 2 is in the vector form, which can be resolved into three components to obtain the scalar equations in x-, y-, and z-directions, given by

____

____ (EQN. 2A)

____

The principle of linear impulse and momentum will also apply to a __________________ __________________. In the case of multiple particles in the system, we can derive the principle of linear impulse and momentum using the _______________ of the momenta and impulses from all particles. Using the same derivation technique as before, we can obtain

______________________________________ (EQN. 3)

From equation3, we can derive another important relationship between impulses and momentum. When the sum of the external impulses acting on the system of particle is zero, we obtain

_______________________________________ (EQN. 4)

which states that the total and final momenta are equal. This equation is referred to as the conservation of linear momentum.

Impact

Examples of impact loadings include the striking of a hammer on nail, or a golf club on a ball. Impact occurs when ____
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____.

There are generally two types of impact, namely ______________________ impact and ____________________ impact.

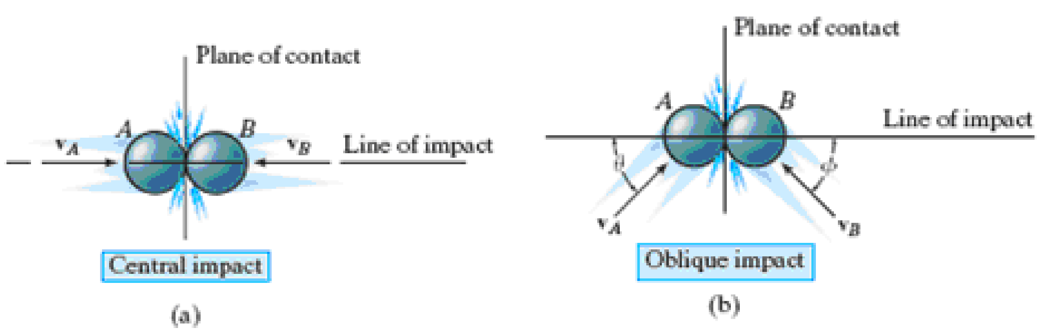

Figure 2

1. Central impact

The central impact is characterised by the ________________________ of the direction of motion of the particles' mass centres with the line of impact. (See figure 2a)

There are several key points about the central impact.

- A central impact will occur if velocities of two particles are _________________, and their magnitudes are not equal

- During the collision, the particles are thought to be _______________________ (nonrigid). During this period of deformation, they exert ___________________ ___________________________ deformation impulse on each other. It is also during the maximum deformation when both particles move with a ___________ _________________________.

- After the maximum deformation has occurred, the particles enter the period of ______________________________ where they return to their original shape (or remain permanently deformed). This restitution impulse causes the particles to depart from each other.

- Normally, the deformation impulse is always greater than that of restitution, i.e. coefficient of restitution is ____.
The coefficient of restitution is the __________________ of the relative velocity of the particles' after the impact to the relative velocity of the particles' approaches before the impact. It is given by

_________________________________ (EQN. 5)

where subscripts 1,2 denote instances before and after impact, respectively
subscripts v_A, v_B denote velocities of particle A and particle B, respectively

A collision is said to be ____________________________________ when the associated coefficient of restitution is $e=1$, i.e. the relative separation velocity is the same as the relative approach velocity of the particles before and after the collision. This cannot be achieved in reality.

A ________________________________ is opposite to the perfectly elastic impact. The coefficient of restitution is $e=0$ in this case. This type of impact is characterised by the particles sharing a common velocity after the impact has occurred, i.e. the two particles are stuck together during the collision.

2. Oblique impact

An oblique impact is characterised by the motion of one or both of the particles is at an angle with the line of impact. See figure 2b. In this case, we have to determine both x- and y- components of the velocity of the particles after the impact.

Angular momentum

The angular momentum $\mathbf{H}_O$ of a particle about point O is defined as the 'moment' of the particle's linear momentum about O. This quantity is sometimes referred to as the moment of momentum.

Using vector, the angular momentum of a particle of mass *m* is given by

__________________________________ (EQN. 6)

where $\mathbf{r} = r_x\mathbf{i} + r_y\mathbf{j} + r_z\mathbf{k}$ denotes a position vector from point O to the particle P

$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$ denotes the particle's velocity

The angular momentum can also be computed using scalar formulation. Suppose that the particle's motion lies in the x-y plane. The magnitude of the angular momentum about the z-axis about point O is, therefore, given by

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where d denotes the perpendicular distance from the origin to the line of action of $m\mathbf{v}$. See figure 3b for illustration.

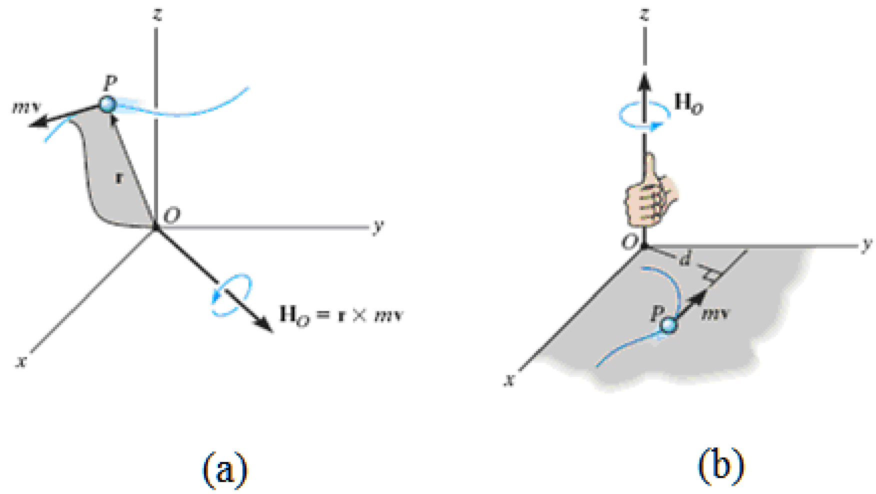

Figure 3

Recall that the relationship between the resultant force $\Sigma \mathbf{F}$ and momentum of a particle is given by

$$\Sigma \mathbf{F} = \dot{\mathbf{L}} = \frac{d}{dt}(m\mathbf{v})$$

where $\dot{\mathbf{L}}$ denotes the time rate of change of particle's linear momentum.

The relationship of the angular momentum $\mathbf{H}_O$ and the resultant moment $\Sigma \mathbf{M}_O$ about point O of a particle takes a similar form, which is given by

____ (EQN. 7)

Note that equation 7 is also applicable to systems of multiple particles

Angular impulse and momentum principles

Let us rewrite and integrate equation 7 between the limits $t = t_1$, $\mathbf{H}_O = (\mathbf{H}_O)_1$ and $t = t_2$, $\mathbf{H}_O = (\mathbf{H}_O)_2$ to obtain the relationship

$$\Sigma\int_{t_1}^{t_2}\mathbf{M}_O\,dt = (\mathbf{H}_O)_2 - (\mathbf{H}_O)_1$$

or

____ (EQN. 8)

Equation 8 is referred to as the principle of angular impulse and momentum. The term $\left[\Sigma\int_{t_1}^{t_2}\mathbf{M}_O\,dt\right]$ is called the angular impulse, hence equation 8 may be interpreted as the initial angular momentum plus the angular impulse must equal the final angular momentum. This form is again similar to the linear momentum counterpart.

Note that the angular impulse is given by

$$\Sigma\int_{t_1}^{t_2}\mathbf{M}_O\,dt = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \qquad \text{(EQN. 9)}$$

where **r** is a position vector which extends from point O to any point on the line of action of force **F**.

Finally, when the angular impulses acting on a particle are zero during the time interval $t_1 < t < t_2$, equation 8 reduces to

____________________________________ (EQN.10)

This equation is known as the conservation of angular momentum.