Lecture 3 – Kinetics of a particle Part 2: Work and Energy

The work of a force

Let us first establish the definition of work. A force, denoted by a vector **F**, is said to do ______________________ on a particle only when the ___________________________ ____.

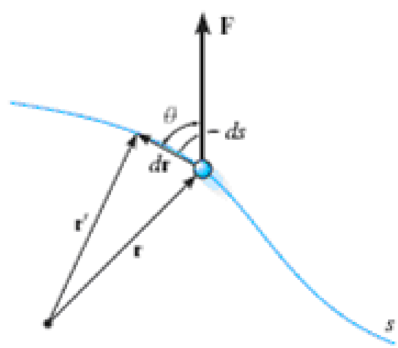

Figure 1

In the figure, the particle moves along the path from the original position **r** to a new position $\mathbf{r'}$. The displacement is given by $d\mathbf{r} = \mathbf{r'} - \mathbf{r}$, whose magnitude is represented by ds. If the angle between $d\mathbf{r}$ and **F** is θ, then the work dU, which is done by the force **F**, is defined by

____

____

Note that the unit of work done is called a ___________________________ By definition, 1 joule of work is done when one Newton of force moves one meter along its line of action.

Work of a variable force

A force is said to be __________________________ when its ___________________ or _________________________ is ___________________________________. The total work done of a variable force can be given by

____

____

Work of a constant force

For the case of a force $\mathbf{F}_c$, which of ____________________________ and it acts at a ____________________________________ to the straight line path. The work done when a particle is displaced from s_1 to s_2 can be determined using

____

____

Work of a weight

If we consider a particle of mass **m**, in a 3D space defined by a set of perpendicular x- y- and z-axes, its weight must be given by a vector $\mathbf{W} = -W\mathbf{j}$, i.e. its direction is always directly downwards. Suppose that a particle is displaced in the same 3D space along the path s from position s_1 to s_2. An intermediate point, the displacement is given by $d\mathbf{r} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}$. The amount of work done by the weight is given by

$$U_{1-2} = \int \mathbf{F} \cdot d\mathbf{r} = \int_{s_1}^{s_2} (-W\mathbf{j}) \cdot (dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k})$$

$$= \int_{y_1}^{y_2} -W \; dy = -W(y_2 - y_1)$$

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Work of a spring force

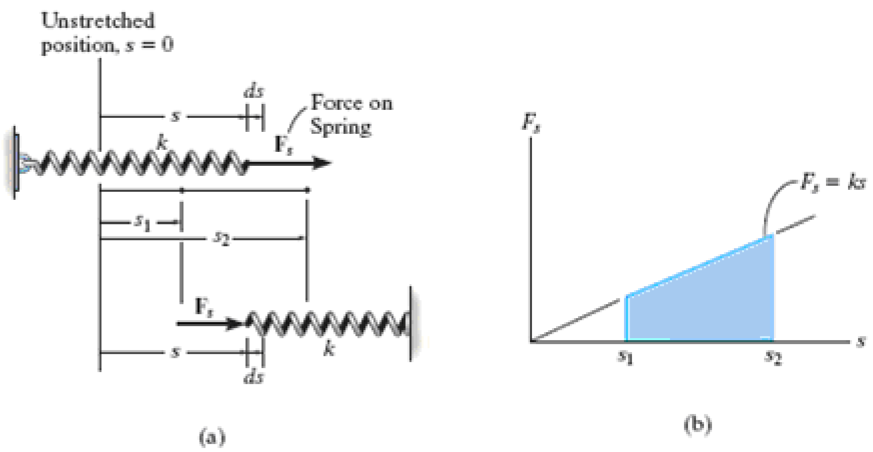

Figure 2

A linear spring of stiffness k provides a restoring force ________________when stretched (or compressed) by a displacement s. The force $\mathbf{F}_s$ which stretches (or compresses) the spring is said to have done some work, as ________________________

____.

Suppose the spring is stretched or compressed by an applied force $\mathbf{F}_s$ from the original position s_1 to a new position s_2, the total work done during this process is given by

$$U_{1-2} = \int_{s_1}^{s_2} F_s \, ds = \int_{s_1}^{s_2} ks \, ds$$

____

Note that the work done by the spring can be graphically represented by the area under the force-displacement curve.

Principle of work and energy

Recall the two main types of mechanical energies namely, ____________________ and ______________________ energies. These will be discussed further in their respective topics in this lecture. One thing of note here is that the SI unit for energy is ___________, which is the same as that used for work done.

Application of principle of work and energy for a system of particles

Consider a particle *P* of mass *m* in figure 3. A resultant force ______________________ (the resultant force represents a system of external forces) is acting on the particle, so that it moves along the path *s*, for example from point s_1 to point s_2.

The equation of motion of the particle in the tangential direction is given by __________. Since the tangential acceleration can be written as ____________________, the equation of motion can be rewritten as

$$\Sigma \int_{s_1}^{s_2} F_t \; ds = \int_{v_1}^{v_2} mv \; dv$$

$$U_{1-2} = \frac{1}{2} m (v_2)^2 - \frac{1}{2} m (v_1)^2$$

____________________________________ (EQN. 1)

where _______________ represents the ______________________________________.

Equation 1 can be rearranged to give ______________________, which states that the particle's initial kinetic energy plus the work done by all the forces acting on the particle as it moves from its initial position to its final position is equal to the particle's final kinetic energy.

This relationship of work and kinetic energy is particularly useful when one wants to determine the final velocity of the particle.

Equation 1 can also be extended to ___________________________________, i.e.

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where the summation represent the total work done and kinetic energy of all particles considered in the system.

Work of dry friction

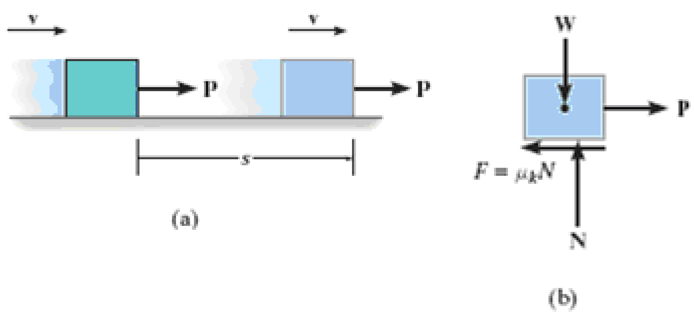

Figure 4

The figure shows a block P of mass m being pulled along a horizontal rough surface whose ____.
The initial velocity is given by v_1 and the final velocity is v_2. We can apply equation 1 to obtain

____

Power

The definition of power states that it is ____
____. Its mathematical expression is given by

____

Recalling that the incremental work done is $dU = \mathbf{F} \cdot d\mathbf{r}$, hence it is possible to derive the following.

$$P = \frac{dU}{dt} = \frac{\mathbf{F} \cdot d\mathbf{r}}{dt} = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt}$$

____

where **v** represents the instantaneous velocity of the point which is acted upon by the force **F**.

The SI unit of power is the ________________ and the unit is defined as

____

The quantity power can be regarded as the capacity to do a certain amount of work in a given time.

Another common unit for power is the horsepower, where 1 horsepower is equivalent to 745.7 W.

Efficiency

Mechanical efficiency of a machine is defined as the _____________________ of output useful power produced to the input power supplied. The mathematical expression for efficiency is given by

____

However, if the efficiency is being measured during the same time interval, we can directly evaluate the efficiency by comparing the amount of energy gained and supplied.

____

Note that some textbooks use the symbol ________________ to denote efficiency

Potential energy and conservation of forces

Conservative force

When the work done by a force in moving a particle from one point to another is ____________________________________ , then this force is considered a conservative force. ____ are examples of conservative forces.

On the other hand, ____ ____, i.e. the longer the path, the greater the work done. These forces are called nonconservative forces.

Potential energy

The potential energy of a particle comes from ____________________________ measured relative to a fixed datum. Recall that kinetic energy is related to the __________________ of the particle.

Gravitational potential energy

The potential energy of a particle of mass m located a distance y vertically above an arbitrary datum is given by

____

where g represents the acceleration due to gravity. It is possible for a particle to have negative potential energy, if its vertical position is below a chosen datum.

Elastic potential energy

When an elastic spring is stretched or compressed by a distance s from the original unstressed position, its elastic potential energy is given by

____

Note that ___________________________________.. This is because the spring always has 'potential' for doing positive work on the particle to bring it back to the original unstretched position.

Potential function

Generally, if a particle is subjected to both gravitational and elastic forces, the particle's potential energy can be expressed as a potential function

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Conservation of energy

When a particle is acted upon by a system of conservative forces only, the total energy, consisting of kinetic and potential energy, of the particle is considered conservative. A mathematical expression showing the conservation of energy of a particle at two instances is given by

____

Let us consider the case of a ball being dropped to the ground. (Neglecting air resistance)

1. It is initially at rest at a height above the ground. Here its potential energy is at the maximum and the kinetic energy is zero.
2. The ball is falling with a certain speed at midway point. Here its potential energy is half its original value and it has already gained some kinetic energy.
3. The ball is about to hit the ground. Here its potential energy is said to be zero and its kinetic energy is at the maximum.

Conservation of energy of a system of particles

If a system of particles is subjected to conservative forces, then the total energy of the system is also conservative. This is represented by a mathematical expression as