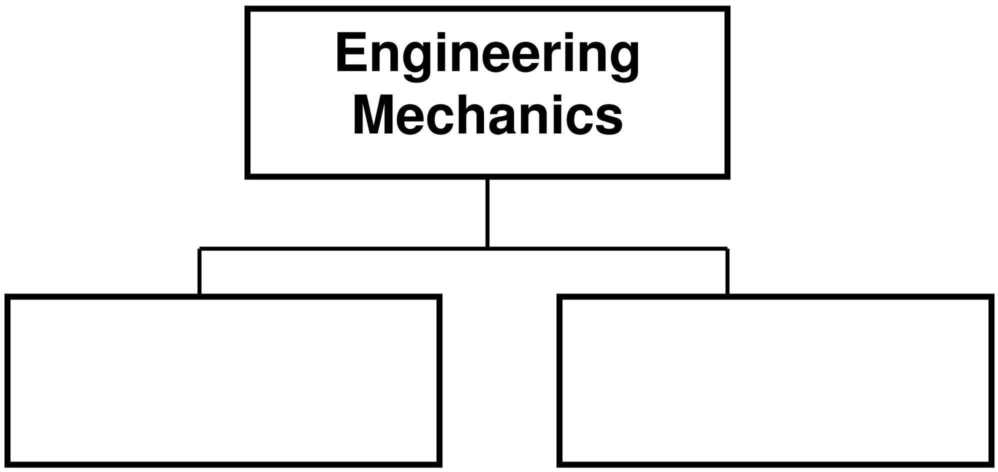
Lecture 1 – Introduction to Engineering Mechanics: Dynamics



Engineering mechanics is the physical science which studies ____________________ ________________________________. The subject is usually divided into two parts namely ____________________________ and _____________________________.

Statics. This branch of mechanics studies the _______________________ of bodies under the action of constant forces and moment, i.e. a body that is either ________________ or ____. Examples of static systems include an aeroplane at cruising speed, a hovering helicopter, a floating stationary ship, etc.

Dynamics. This branch of mechanics studies the __________________________________, i.e. system where a body is acted upon by an externally applied force which is a function of time. Examples of dynamic mechanical systems include a grandfather clock pendulum, a mass-spring system, an accelerating/decelerating vehicle, etc.

Background Knowledge

- Newton's second law of motion states that ________________________________
____
____.

- The study of mechanical dynamics involves the effects of the forces on a body, and the motion that ensues. The subject can be generally presented in two parts
    1. Kinematics, ____.
    2. Kinetics, ____.

A dynamic mechanical system: a 1-DOF mass-spring system

The system in figure 1 consists of a block of mass M which is free to move horizontally on a frictionless surface. The block is attached to a spring whose stiffness is k, and x denotes the displacement of the block in the horizontal direction and is measured relative to a datum point. The spring is assumed to be unloaded when $x=0$. Note that the displacement x takes a positive value in the direction of the arrow in figure 1 (towards the right hand side).

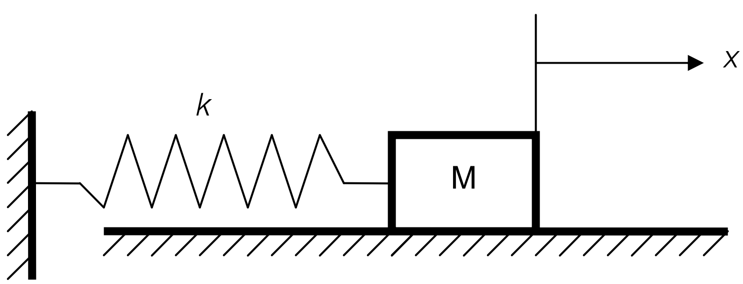


Figure 1

In this example, we will consider the following actions being performed on the block

1. The block is originally at rest.
2. A force G is applied on the block so that it moves towards the right until the displacement is $x=x_1$.

3. The force is removed and the block is oscillating under the influence of the spring stiffness.

The following describes the force analysis of the system at various stages.

Step 1

If we perform a force analysis of the system when the block is at rest, then we will have a ______________________. The spring is unstretched, hence it is not applying any force on the block. The weight *W* of the block is supported by the reaction force *R* from the ground.

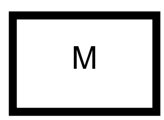


Figure 2

Step 2

When a horizontal force *G* is applied, the block will travel in the direction of the force for a distance x_1. At this point, the block is ____ as the force *G* is now balanced by the tension $T = kx_1$ in the spring and the weight is supported by the reaction from the ground. Hence, the resultant force acting on the block is _____________________________________.

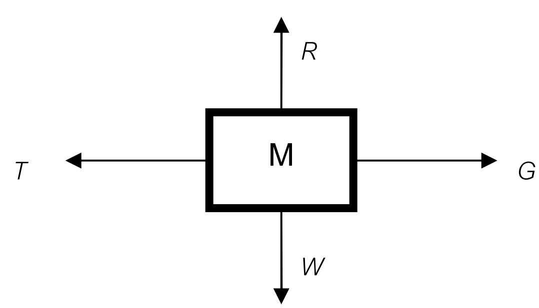


Figure 3

Note that we are neglecting the motion of the block while it is travelling between $x=0$ and $x=x_1$.

Step 3

The force G is suddenly removed and the block is now experiencing ________________ ___________________ exerted by the spring. This unbalanced force will result in a ___________________________________ and this motion can be analysed using a relationship suggested by Newton's second law of motion.

________________________________ (Eqn. 1)

F denotes the total unbalanced (resultant) force acting on the block

m denotes the mass of the block

a denotes the acceleration of the block in the direction of the force

Now we can use equation 1 to determine the motion (or kinematics) of our block. However, before we can start analysing the motion of the block, it is necessary to establish a way to do so.

The motion of the block can be represented by a time function of its displacement, i.e. $x(t)$, where t denotes the time.

The velocity of the block, denoted by v, is simply a time derivative of the displacement, hence it can be written as

____ (Eqn. 2)

The acceleration of the block, denoted by a, is a time derivative of the velocity of the block. Alternatively, it is the second time derivative of the displacement. Its mathematical expressions are given by

____ (Eqn. 3)

Here, we will use SI units for all our calculations. The units for time, displacement, velocity and acceleration are given in table 1.

Quantity	**Unit (SI)**	
Time	second	s
Displacement	metre	m
Velocity	metre per second	$m\ s^{-1}$
Acceleration	metre per second	$m\ s^{-2}$

Table 1

Let us now consider equation 1 again. The resultant force F acting on the block is entirely due to the spring stiffness which is given by

____

Note that the direction of the restoring force exerted by the spring is always __________________.

The mass of the block is given by m, and the acceleration of the block is $\ddot{x}$. Hence, equation 1 can be rewritten as

____ (Eqn. 4)

which is a second order differential equation in x. By inspection, the solution of the differential equation must be a _______________________________ function. In this case, we will use a sine function, so the displacement, velocity and acceleration of the block are given by

____ where $\omega = \sqrt{\frac{k}{m}}$ and $\hat{x} = x_1$. The quantity ω also denotes the ________________________ of the motion (oscillation), and $\hat{x}$ denotes the ___________________________________.

The type of oscillation which can be described by a trigonometry function, such as that in equation 5, is called simple harmonic motion.

The solution of the dynamic mechanical system is given by equation 5. It describes the kinematics of the block by showing the displacement *x* as a function of time *t*. Alternatively, this can be graphically represented by a time history plot.

Displacement, Velocity, and Acceleration

This section will examine the relationship between the displacement, velocity and acceleration. They are time derivatives of one another as already explained earlier.

Let us consider equation 2 $\left[v = \frac{dx}{dt} = \dot{x} \right]$. If we draw a velocity-time graph, the value of the slope indicates ____ while the area under the curve (between limits) ____.

We can integrate the above equation to obtain

$$x = x_0 + v_0 t$$

where x represents the displacement of the motion. x_0 and v_0 are initial values of displacement and velocity, respectively.

Therefore, if we draw displacement-time graph, the slope of the curve shows the ____.

Projectile Motion

A projectile motion can be defined as one with a _________________________________ ______________________________, hence a classic example of a projectile motion is the free-flight of a thrown object which follows ___________________________________ ______________________. The retardation due to air resistance is considered negligible.

The mathematical expressions of accelerations, velocities and displacement in the vertical (y) and horizontal (x) directions are given by

____

____

____

General motion of a particle in 3-D space

Cartesian coordinates

The simplest representation of the position of a particle in a 3D space is by using the Cartesian coordinates, which utilises ____.
The position is defined relative to a stationary datum point, usually the origin, by a three-component vector **r**, which is given by

____

where **i, j**, and **k** are unit vectors in the direction of (positive) x-, y-, and z-axes, respectively. This is illustrated in figure 1.

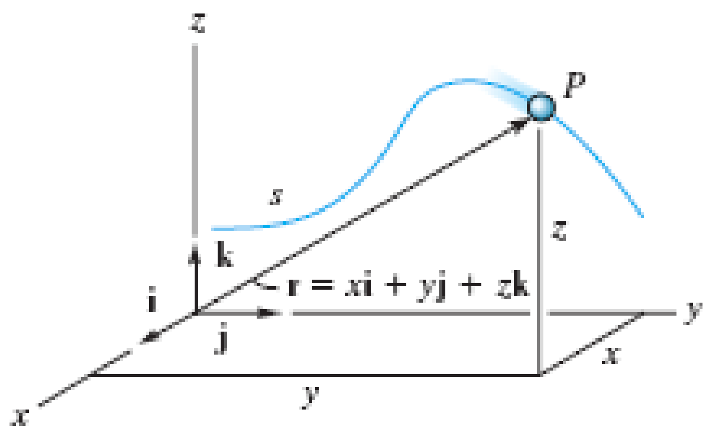


Figure 1

Recall that the velocity and acceleration are first order and second order time derivatives of the displacement, hence these are given by

____

____

Cylindrical coordinates (Polar coordinates in 3D)

Usually the polar coordinates are used in 2D applications when an analysis of particles ____ is involved. This can be extended to 3D by including a________________________ vector, as shown in figure 2.

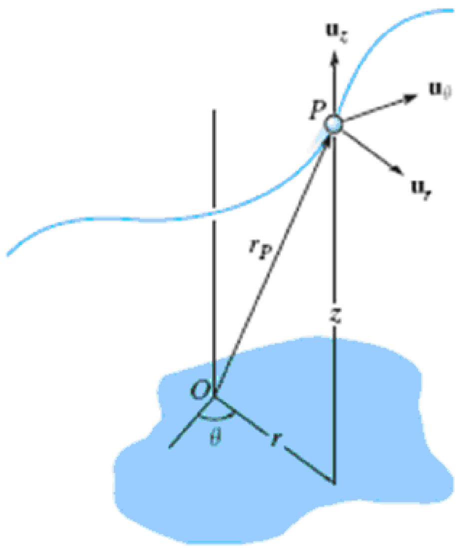


Figure 2

The position vector is given by

____

where $\mathbf{u}_r$ is the unit vector in the radial direction of the cylinder, and

$\mathbf{u}_k$ is the unit vector in the direction of the height of the cylinder

The velocity and the acceleration of a particle in polar coordinates are given by

____

____

Relative motion analysis

The relative position of B with respect to A, $\mathbf{r}_{B/A}$, is given by

____

Similarly, the relative velocity and acceleration of particle B with respect to particle A are given by

____

____