Lecture 6: Distributed Forces Part 2 – Second Moment of Area

The second moment of area is also sometimes called the ______________________. This quantity takes the form of

____

The physical representation of the above integral can be described as follows.

When _________________________ are distributed continuously over ____________, the ______________ of these forces about some axis is proportional to the distance of the line of action of the force from the moment axis. Considering an elemental area on the surface, the elemental moment is proportional to the distance squared times the elemental area.

Example

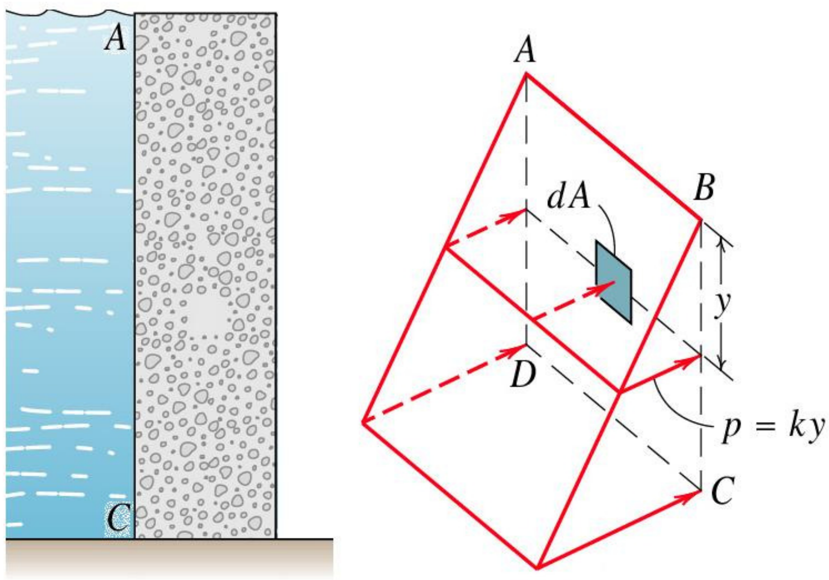

Figure 1

A reservoir wall denoted by a surface *ABCD* is holding water on the inside. The water applies a ______________________________ which increases linearly with the depth on the wall. The moment about the axis *AB* due to the pressure on the element of area *dA* is given by

$$dM = (py)dA = (ky^2)dA$$

Hence, the total moment over the surface *ABCD* can be found by integrating the above elemental moment over the area. This is given by

$$M = k\int y^2 \; dA$$

Definitions of second moment of area

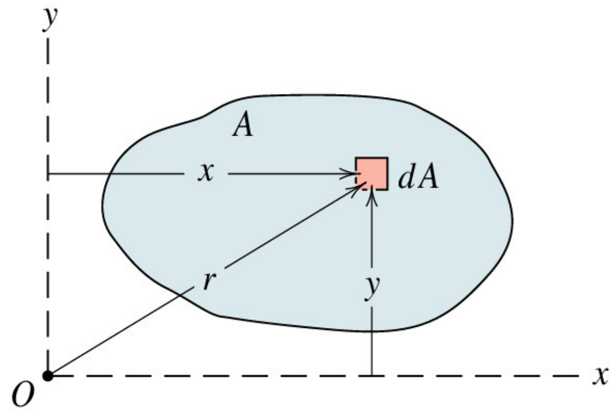

Figure 2

Figure 2 shows an area *A* in the *x*-*y* plane with a small element of area *dA*. By definition, the second moments of area of the element about the *x*- and *y*-axes are given by $dI_x = y^2 dA$, and $dI_y = x^2 dA$, respectively.

Therefore, the second moments of area of the entire area A about the *x*- and *y*-axes are defined as

____

____

Consider a rectangular lamina as shown in figure 3. We will determine the following

1. __________________________ (Example 1)
2. __________________________ (Example 2)

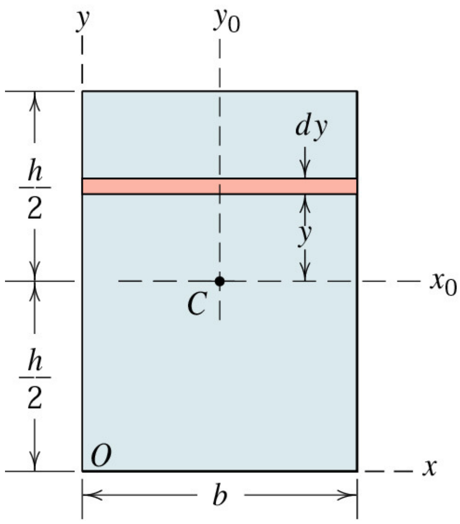

Figure 3

Example 1

We will calculate the moment of inertia about the x-axis, I_x, which lies along the bottom edge of the lamina. Let us define the elemental area *dA* using horizontal strip elements as shown in figure 3, so that

____

The integration begins ______________________________ of the lamina and finishes at ___________________________, so the integration limits are ________________, respectively. Finally, the moment of inertia about the x-axis is given by

____________________________ (1)

Similarly, the moment of inertia about the y-axis is found using vertical strip elements so the elemental area is given by

____

The integration is now bounded between *0* and *b*, therefore the moment of inertia about the y-axis is given by

_______________________________ (2)

Example 2

Let us now shift the x- and y-axes from the corner of the lamina to _______________. From inspection, the centre of the rectangular lamina also coincides with its centre of gravity. Usually, moments of inertia taken about the x- and y-axes through the ___________________________ are written as ____________________, respectively.

The moment of inertia about the horizontal axis labeled x_0 in figure 3 is given by

______________________________ (3)

Similarly, the moment of inertia about the vertical axis y_0 is given by

______________________________ (4)

Parallel Axis Theorem

Note that equations (1) and (3) are very similar. Only their integration limits are different. The difference in their physical meaning is the ________________________ ___________________ from x_0-y_0 (axes through the centroid).

Therefore, moments of inertia of the same body taken about axes _________________ can be related using

____

____

where A is the area of the lamina

d_x is the ________________________ between x_0- and x-axes

d_y is the ________________________ between y_0- and y-axes (See figure 4)

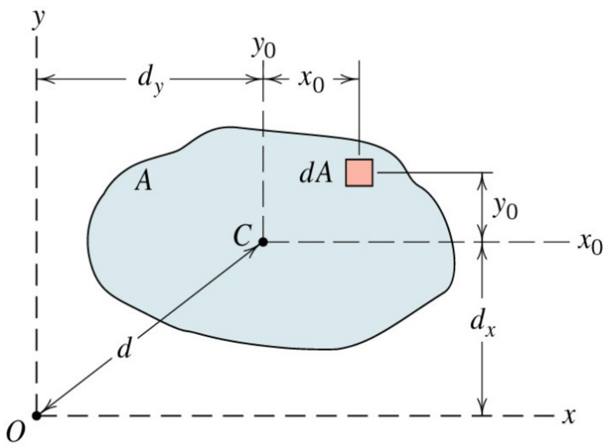

Figure 4

Radius of Gyration

The radius of gyration for the x-axis, ________$_x$, is defined by the relationship

__________________ or __________________

where A is the area of the lamina

I_x is the second moment of area about the x-axis (horizontal axis)

Consider an irregularly shaped lamina of area A in figure 5A. Its moment of inertia about the x-axis is given by I_x. Now, consider a long thin (negligible width) strip in figure 5B whose area is also A. The moment of inertia of the thin strip in figure 5B is also given by I_x. The distance k_x is known as the radius of gyration.

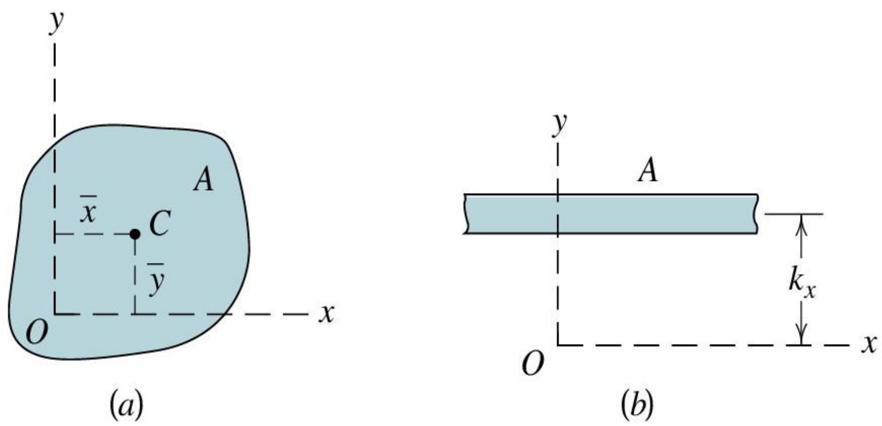

Figure 5

Similar expressions for moments of inertia about the y- and z-axes are given by

$$I_y = k_y^2 A \qquad \text{or} \qquad k_y = \sqrt{I_y / A}$$
$$I_z = k_z^2 A \qquad \qquad k_z = \sqrt{I_z / A}$$

Perpendicular Axis Theorem

Suppose we need to determine the moment of inertia about the z-axis, I_z, of the rectangular lamina shown in figure 3. Let us define the z-axis to be perpendicular to the plane which contains x- and y-axes.

We can make use of known quantities found in equations (1) and (2) to determine the moment of inertia about the z-axis. The perpendicular axis theorem states that

____

This relationship is useful when determining the moments of inertia of a lamina about an axis which is perpendicular to its surface.

Composite Bodies

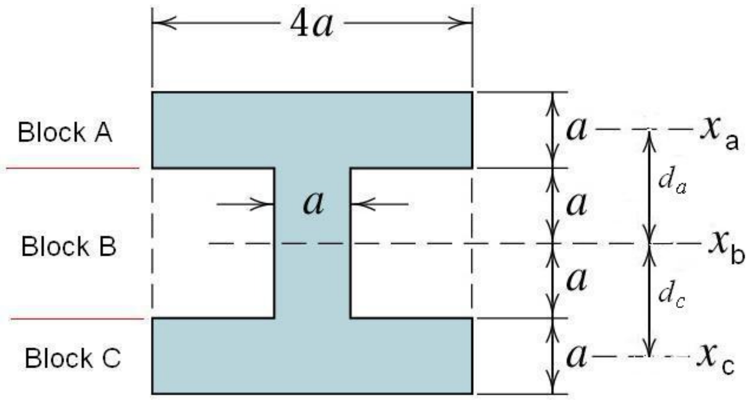

Figure 6

Figure 6 shows the cross section of an I-beam, which can be easily disassembled into three parts, block A, block B and block C, respectively. The moment of inertia of the entire shape about the central axis (x_b axis) can be found by ______________________
____ using the relationship

____

where d represents the perpendicular distance between the axes.

In general, the moments of inertia of a composite area about the x- and y-axes are given by

____

____