Lecture 5: Distributed Forces Part 1 – Centre of Mass

Figure 1(a)

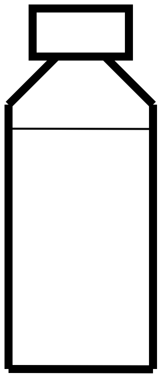

Figure 1(b)

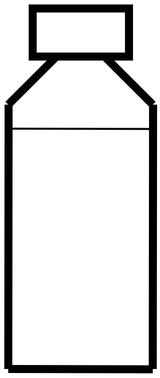

Figure1(c)

Figure 1(a) shows a bottle of water being held aloft at its cap. The fingers are applying pressure normal to the surface of the cap while the friction between the cap and the fingers is equal and opposite to the total weight of the volume of water in the bottle.

This mechanical system is shown in ______________________________ in figure 1(b), where the pressure applied by the fingers and the weight of the water are illustrated by ____________________________.

Compare the system to figure 1(c), a ________________ which was used before in a previous lecture. The forces are assumed to __________________________________. The entire weight of the water in the bottle seems to act at ______________________ ____________________. This point is its __________________________________.

The simplification shown in figure 1(c) is a very common tool in engineering application. This is because the force at the contact between two objects almost never acts at a particular point, but over a finite area however small.

Centre of mass

The centre of mass is defined as ____

____.

The centre of gravity is defined as ____

____. For the purpose of this lecture, the centre of gravity and the centre of mass are the same and can be used interchangeably.

A simple experiment to determine the centre of mass of an arbitrarily shaped _______ (objects of constant thickness and mass, i.e. sheets of uniform materials) may be set up as shown in figure 2.

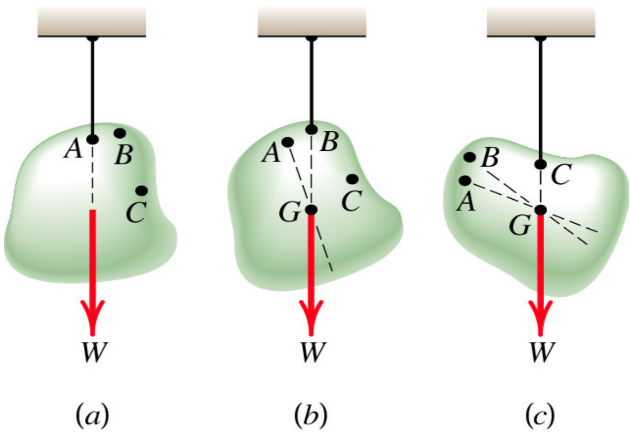

Figure 2

The centre of mass of a lamina may be found by hanging it by the small holes at different positions. The weight of the lamina must act through the centre of mass which must be vertically directly below the hole. By repeating the process several times, the point where the lines of action intersect is the position of the centre of mass.

Centre of mass of a geometric body

Here we will concentrate on the centre of mass of two-dimensional bodies of geometrical shapes, i.e. circles, rectangles, triangles, etc. The same approach may be used for one- and three-dimensional bodies.

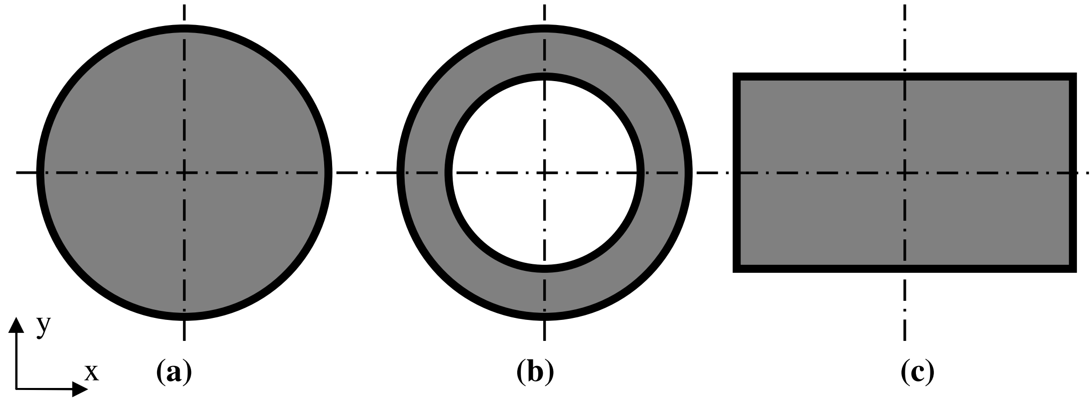

Figure 3

From inspection, the centres of mass of all the shapes in figure 3 must lie in the centres. This is because they are all symmetric about the centre lines in the x and y directions.

Position of centre of mass of a composite geometric 2-D body

For a body which consists of geometric parts such as one shown in figure 4, the position of its centre of mass may be determined by ____________________________

____.

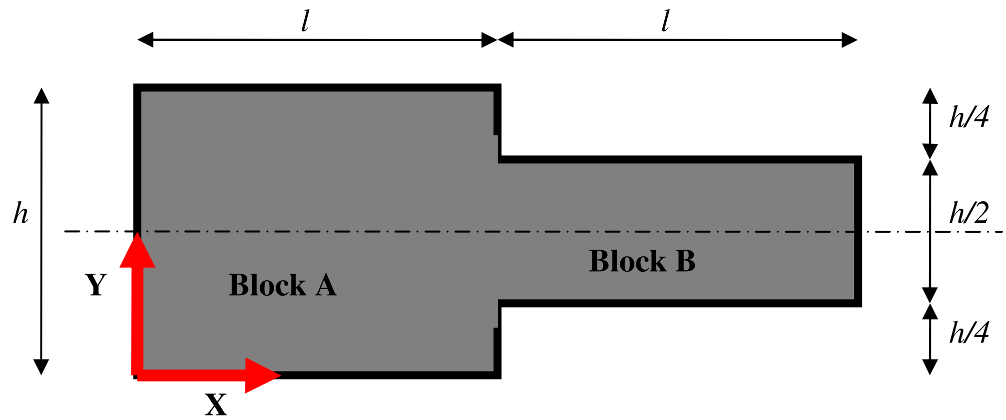

Figure 4

The coordinates of the centre of mass of block A are_____________ and ___________. Similarly, the coordinates of the centre of mass of block B are _____________ and

__________________.

From inspection, the centre of mass of the entire body must lie on the horizontal centre line, i.e. ________________, and its x-coordinate should be between those of blocks A and B.

Imagine that the block is being supported on the left and on the centre line as shown in figure 5.

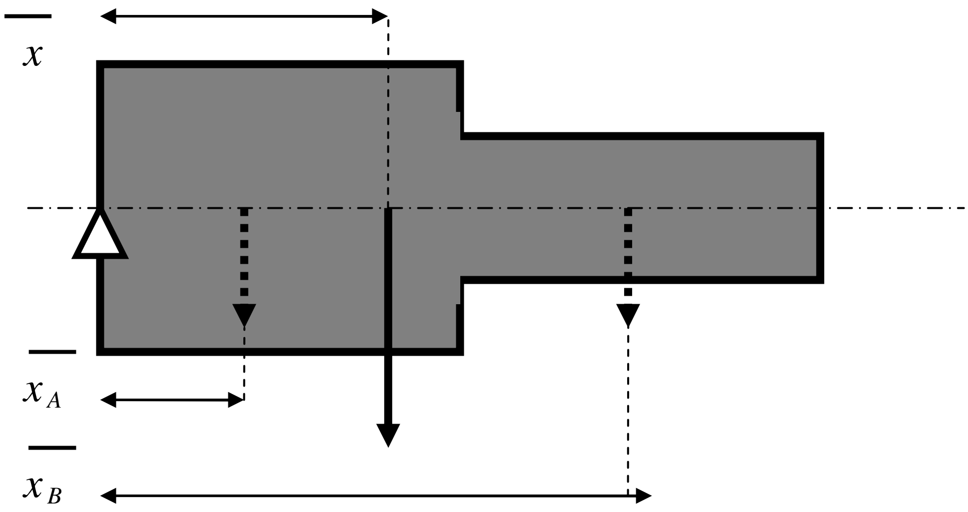

Figure 5

Consider

1. the moment generated by the weight (mass) of the ___________________ acting at the centre of gravity at a distance $\bar{x}$ from the pivot
2. the moments generated by the ________________________________

The moments generated by the two cases must be equal, hence we can derive

$$\boxed{\bar{x}\sum_{i}^{n} M_i = \sum_{i}^{n} \bar{x}_i M_i} \qquad \textit{eqn.(1)}$$

$\bar{x}$ (mass of the entire body) = $\bar{x}_A$ (mass of block A) + $\bar{x}_B$ (mass of block B)

____

Position of centre of mass of a generic 2-D body

For a generic two-dimensional body (lamina), the position of its centre of mass cannot be easily found using the symmetry. Consider a ______________________ shaped lamina shown in figure 6

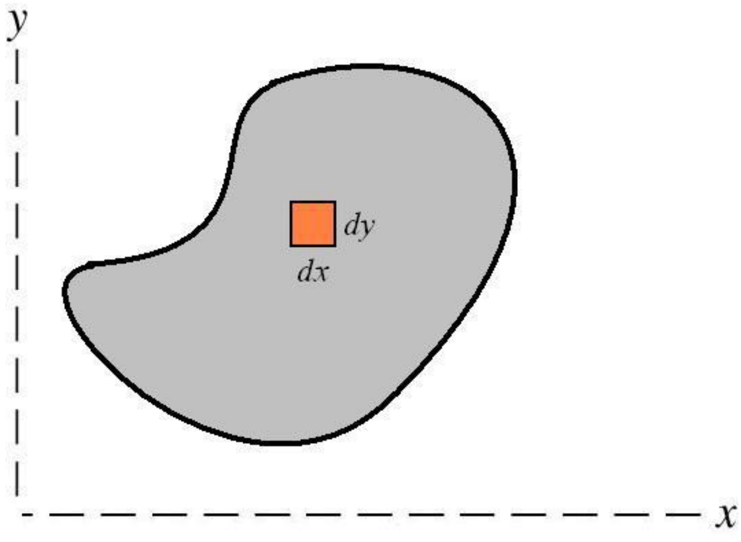

Figure 6

where A represents the area of the shaded region

dA represents the area of the small rectangular element, in this case $dA = dy \times dx$

Recall equation (1) and using the same approach as before (lamina consists of many small elements), the x-coordinate of the centre of mass of the lamina is given by the expression

$$\bar{x}A = \sum_{i}^{n} \bar{x}_i dA_i$$

For an infinitesimally small element of area dA, the expression becomes

______________________ *eqn. (2)*

Similarly, the y-coordinate and the z-coordinate are given by

______________________ *eqn. (3)*

______________________ *eqn. (4)*

Example 1: Centre of mass of a triangular lamina

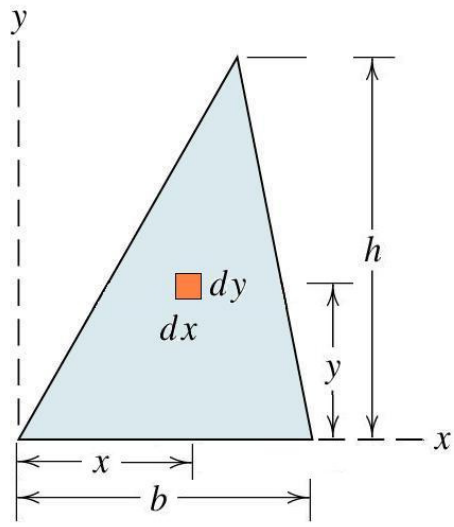

Figure 7

This example will demonstrate the determination of the coordinates of the centre of mass of the triangular shaped lamina shown in figure 7.

1. The area of the triangle is given by ________________________.
2. The area of the element is _________________________.
3. The coordinates of the element are given by ____ and_____.
4. The integration limits are given by

	Lower limit	Upper limit
x direction		*: using the principles of similar triangles*
y direction		

5. Using equation (3), the y-coordinate of the centre of mass is given by

$$\bar{y}\left(\frac{1}{2}bh\right) = \int_0^h \int_0^{\frac{b}{h}(h-y)} y \; dxdy$$

$$\bar{y} = \left(\frac{2}{bh}\right)\int_0^h y\left(\int_0^{\frac{b}{h}(h-y)} dx\right)dy$$

$$\bar{y} = \left(\frac{2}{bh}\right)\int_0^h y\frac{b(h-y)}{h}dy$$

$$\bar{y} = \left(\frac{2}{bh}\right)\left(\frac{bh^2}{6}\right) = \frac{h}{3}$$

6. The centre of mass of the triangle is one-third the height of the triangle measured perpendicularly from the base. This result also holds if the calculation is made with other sides being horizontal, thus the centre of mass of a triangle lies at the intersections of the medians.

Hints and Tips

The ________________________________ which is a result of__________________, may be very difficult to compute. A simplification could be made so that the integration is less daunting. For example, the elemental area of the triangle used earlier could be changed to ______________________________ as shown in figure 8.

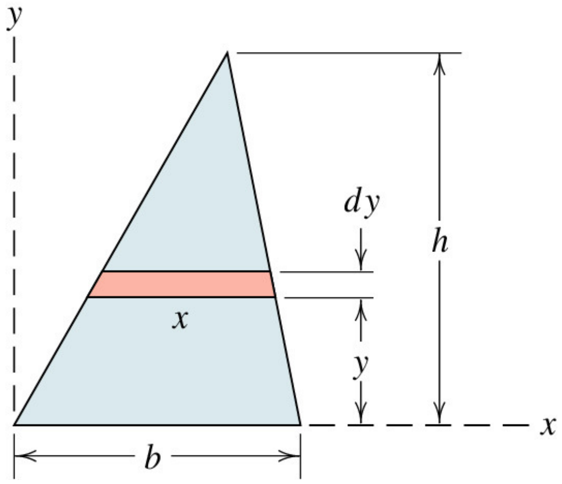

Figure 8

The area of the strip element is given by__________________, where____________. Therefore, the y-coordinate of the centre of mass can be given as

$$\bar{y}\left(\frac{bh}{2}\right) = \int_0^h y \frac{b(h-y)}{h} dy$$

which can be solved straight away.

Example 2: Centre of mass of a circular segment

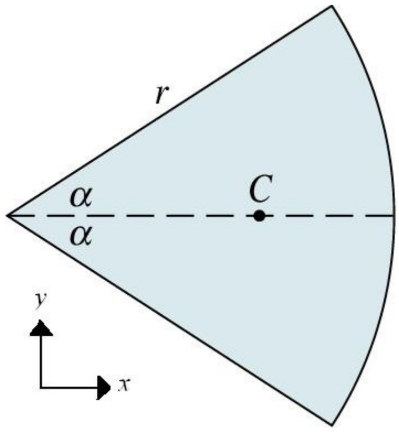

Figure 9

A segment of a circle with radius *r*, and angle α is shown in figure 9. The area of segment is given by__________________. In order to determine the centre of mass of the segment, it is not advisable to use the Cartesian coordinates (*x* and *y* directions), i.e. $dA = dxdy$. As the integration limits become very difficult to define.

We will use the ________________________________ in this case. Consider the strip elements shown in figure 10.

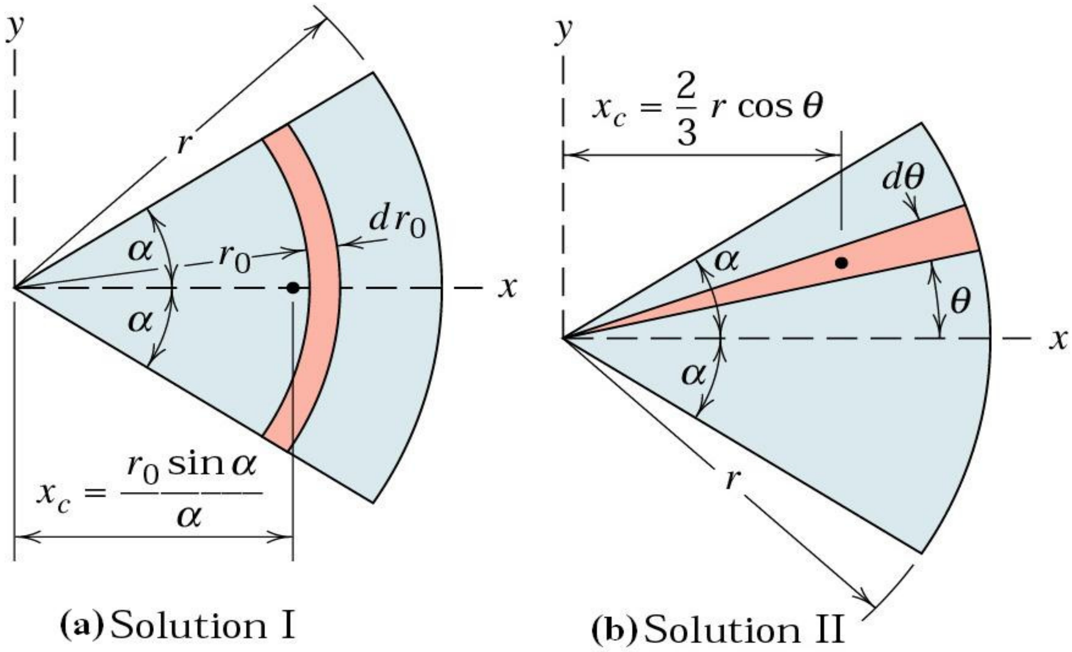

Figure 10

Case (a)

The entire segment is divided into _______________ of thickness______whose area is, correct to first order, given by

____

The centre of mass of the curved strip is given by $x_c = \dfrac{r_0 \sin\alpha}{\alpha}$. (See Meriam page 234 for derivation)

Case (b)

The segment is divided into ____.
Each small segment is given by

____

The centre of mass of the small segment is given by $x_c = \dfrac{2}{3} r \cos\theta$.