## Lecture 3: Structures (Frames and Trusses)

So far we have only analysed forces and moments on a single rigid body, i.e. bars. Remember that a structure is a formed by $\qquad$ and this lecture will investigate forces in such structures.

A $\qquad$ composed of $\qquad$ joined at their ends to form a
$\qquad$ is called a $\qquad$ .


Commonly Used Bridge Trusses


Commonly Used Roof Trusses
Figure 1 - Examples of different types of trusses

## Members of a structure

We will concentrate on structures consisting of $\qquad$ .

These bars are also called members. In a large structure, there may be hundreds or thousands of members and they all carry loads of different magnitudes and directions.

Load carrying members can be classified into three categories

1. Compression

2. Tension
3. Unloaded


Note: There is no specific sign convention for the directions of forces, however it is very important that a consistent system is used throughout the calculation. Here, we will take compression forces to be positive and tension to be negative.

## Method of Joints

Recall that a pin joint connects two or more members (bars) together. The joint resists
$\qquad$ but allows $\qquad$ ,
i.e. the joint only exerts forces but not moment.


Figure 2

Figure 2 shows an example of a pin jointed frame whose properties are given in the table below. Note that $F$ is an externally applied force which acts on the structure at point $C$.

| Number of | Quantity | Names |  |
| :---: | :---: | :---: | :---: |
| Members |  |  | $\mathbf{M}=$ |
| Reactions |  |  | $\mathbf{R}=$ |
| Joints |  |  | $\mathbf{J}=$ |

Table 1

## Example of framework analysis

Here, we will attempt to determine all the forces in the structure shown in figure 2.

## Step 1

The reaction forces must be determined. There is no externally applied force in the horizontal direction, therefore we can deduce that the reaction $R_{l}=0$. The vertical reaction forces $R_{2}$ and $R_{3}$ can be given as functions of the externally applied force $F$. Step 2

Consider joint $A$ and the members connected to it, namely $A B$ and $A C$. The joint is in equilibrium and therefore the resultant force at the joint must be zero. By resolving the forces into horizontal and vertical components and using two equilibrium equations in both directions, the two unknown ( $A B$ and $A C$ ) can be found.

## Step 3

At joint $C$, the vertical equilibrium equation will give the load in the member $B C$, since the load in $A C$ is already known. The load in member $C D$ is given by the horizontal equilibrium equation.

Step 4
Finally, the load in member BD can be found by applying the equilibrium condition at joint $B$.

This analysis is based on $\qquad$ , therefore this method is also known as the $\qquad$ .

## Static determinacy

In the previous part, we were able to determine the loads in $\qquad$ members of the structure. When such calculation is possible, the structure is said to be $\qquad$
$\qquad$ . The condition for a structure to be statically determinate is given by
where $\quad M$ represents the number of members
$R \quad$ represents the number of reactions
$J$ represents the number of joints

For a system where $\qquad$ , such a structure is said to be
$\qquad$ This is because there are more unknowns than available equilibrium equations and one or more members will be indeterminate. Such a system is also called a $\qquad$ system because it has $\qquad$ .

A system where $\qquad$ is called an $\qquad$
$\qquad$ . This is not normally found in standard applications because they are flexible and collapsible under loads.

Examples of redundant frameworks are shown in figure 3.

(a)

(b)

(c)

(d)

Figure 3

## Method of Sections

Previously, we resolved the forces in a framework by the $\qquad$ ,
where the forces around a joint are computed using the equilibrium conditions. An obvious disadvantage of this method is that many calculations will be required if the member that needs to be analysed is far from the reaction forces.

## Example 1

Consider the member $F E$ in the framework in figure 4 . The force in this member can be computed by the method of joints, i.e. $\qquad$


Figure 4
Here, we will introduce the $\qquad$ This will help in quickly determining the force in the member in a structure without having to follow the steps shown before. The following describes how the method of sections can be used to determine the force in $\qquad$ in figure 4.

Step 1
Make an $\qquad$ .

In this case, the structure is cut vertically through the members $F E, B E$ and $B C$.

Step 2
Draw $\qquad$ of the two separate sections with appropriate forces. Each section should be treated as $\qquad$ .
The free body diagrams for the two sections are shown in figure 5.


Figure 5

The arrows indicating the direction of the forces in the diagrams show that member $E F$ is in tension while members $B E$ and $B C$ are in compression. This structure is considerably simple and the direction of the forces can be quite easily predicted.

Note that an incorrect direction of the force (tension or compression) in the cut member does not result in an incorrect calculation. It will only lead to the final value of the force being $\qquad$ .

The most important step while making a cut is that the forces in the cut members
$\qquad$ at the cut on either side of the sections, i.e. the force $E F$ on the left section (towards $F$ ) must be equal and opposite to the force $E F$ on the right section (towards $E$ ).

## Step 3

The required force $E F$ can be computed by $\qquad$
$\qquad$ . This will eliminate forces $L, B E$ and $B C$ as their moment arms are zero, leaving only forces $R_{l}$ and $E F$.

The force $E F$ is given by the expression

## Example 2

Using the framework in figure 4, how would you make a cut to separate the structure into two separate sections in order to determine the force $B C$ ?

