Lecture 2: Force Systems

Part 1: Two-dimensional force systems

Forces are _______________________ quantities. It is very important to understand the basics of vector calculations.

Types of vectors

1. ________________________________. The position of a free vector is not restricted to any unique line in space, ie. velocity vector may be positioned anywhere on the body of an object moving in a straight line.
2. ________________________________. The vector can be applied anywhere on a unique line in space, ie. a force vector applied on a ________________ body can be placed anywhere along the line of action.
3. ________________________________. The vector must be applied on a particular unique point, ie. a force vector applied on a ___________________ body.

Vector quantities can be added or subtracted. Consider the following system of vectors.

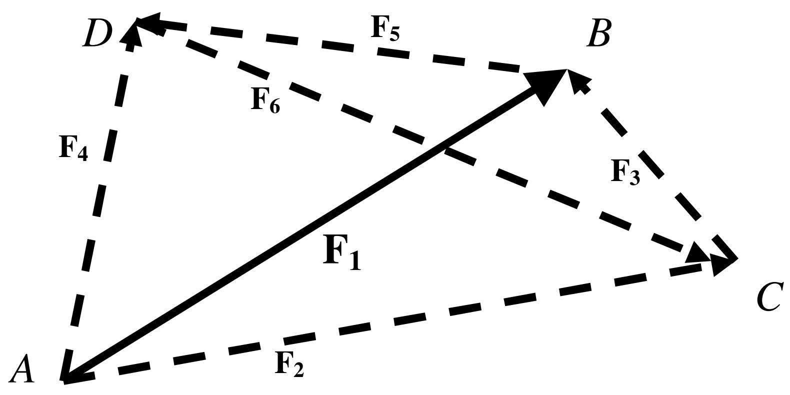

Figure 1

The system shows six vectors namely F_1 to F_6. Vector F_1 connects point A to point B in the direction towards point B. The magnitude of vector F_1 is represented by the length of the vector. The directions and magnitudes of other vectors are illustrated in the figure.

Addition

From figure 1, the vector F_1 begins at point A and is terminated at point B. Now, consider the pair of vectors F_2 and F_3, which also begin at point A and are terminated at point B. This can be mathematically represented as

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Subtraction

The inverse of a vector is defined as a vector quantity of equal magnitude but is acting in the opposite direction.

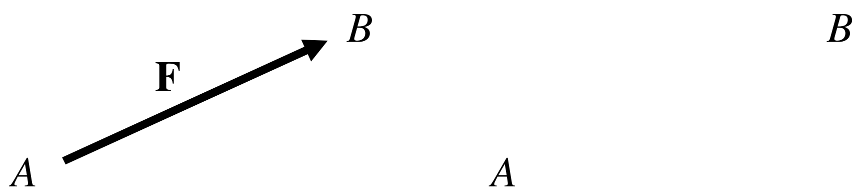

Figure 2

Vectors F_4 and F_5 in figure 1 also connect points A and B, hence the following relationship can be derived.

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Resolving forces

Forces are vectors quantities and can be __________________________________ or _____________________________________ into several components. The following figure shows that force **F** can be broken down into two components namely the ___________________________ and the ________________________________ parts.

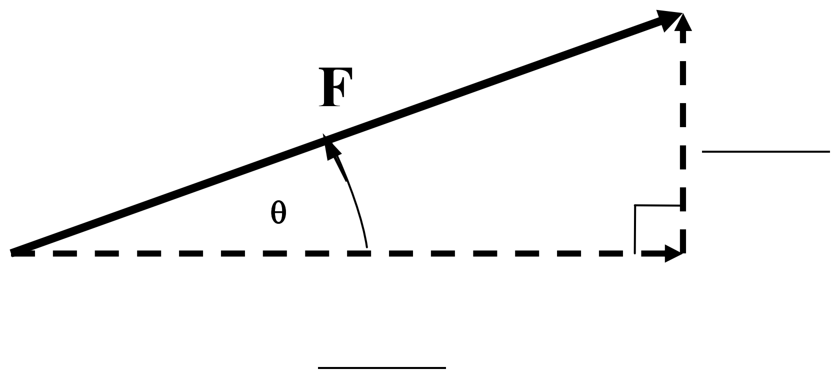


Figure 3

Note that the force F and its components form a right angled triangle. This is very useful because we can use the ________________________________ theorem to determine the ___________________________ of each component.

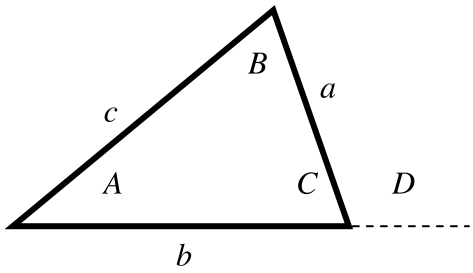
For a generic triangle of forces, the sine and cosine rules can be used to determine the magnitude instead.

Sine rule ________________

____

Cosine rule

____



Resultant (combined) forces

When more than one force is applied, their ________________________________ is equivalent to the effect created by one ________________________________. Consider the system of forces in figure 3.

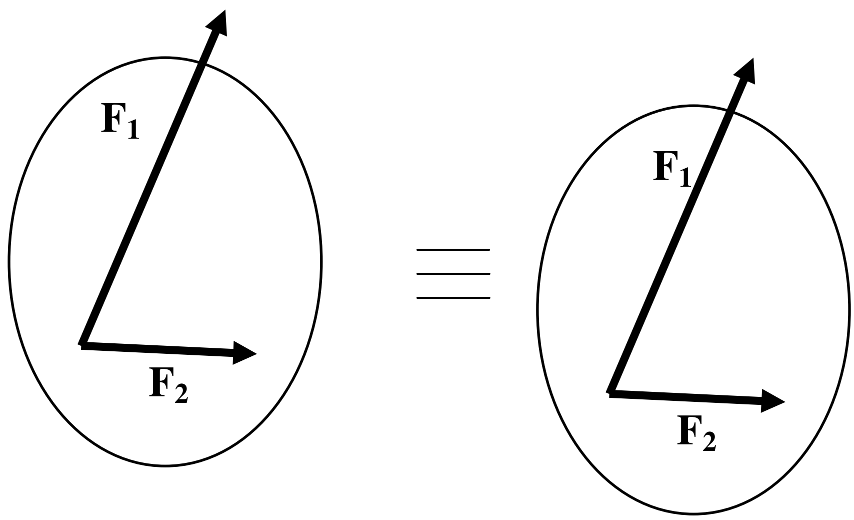


Figure 4

By drawing the system of forces in ______________________________________ or ____, the total effect of forces F_1 and F_2 can be represented by the resultant force R, where

____

Moment

Moment is generated on a body when a force is exerted and its ___________________ ___________________________________. The magnitude of the moment is given by

Moment = _________________ *X* __________________________________

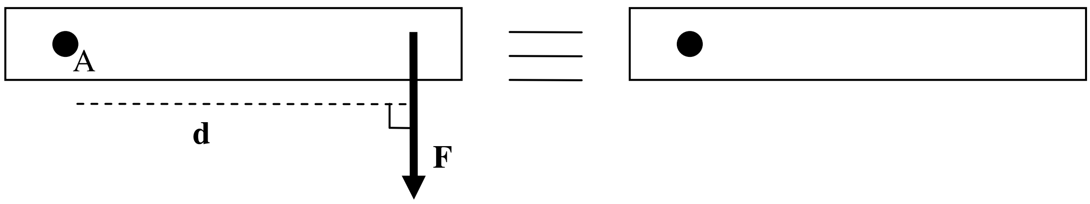


Figure 5

In figure 5, the bar is __________________ at point *A*, i.e. it can ________________ _________________. A vertical force *F* is applied at a horizontal distance *d* from the pivot. This force generates a moment of magnitude *Fd* about point *A*.

Equilibrium

An object is said to be in ____________________________________ when there is ____ acting on it. That is, according to Newton's first law, the body will ______________________________________ or continue to move with a ____.

Newton's laws of motion state that

1. A particle remains at rest or continues to move with a constant velocity unless an unbalanced force acts on it.
2. Force = mass x acceleration
3. The forces of action and reaction between bodies are always equal in magnitude, in the opposite directions and collinear (lie in the same line in space).

From the first law, it can be deduced that an object in equilibrium must have zero resultant force and zero moment acting on it. For a general case of an object in a three dimensional space, the conditions for equilibrium can be given as

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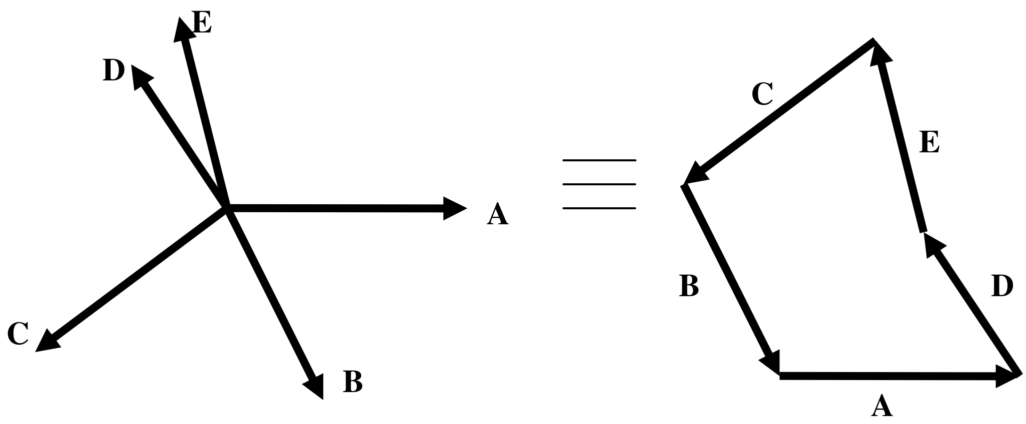
Graphically, all forces in a system in equilibrium must form a closed loop.



Force-couple systems

When a pair of forces of ____,

____, and ____________________________

(parallel but do not coincide) are exerted on a body, the moment they generate is

called a __________________________________.

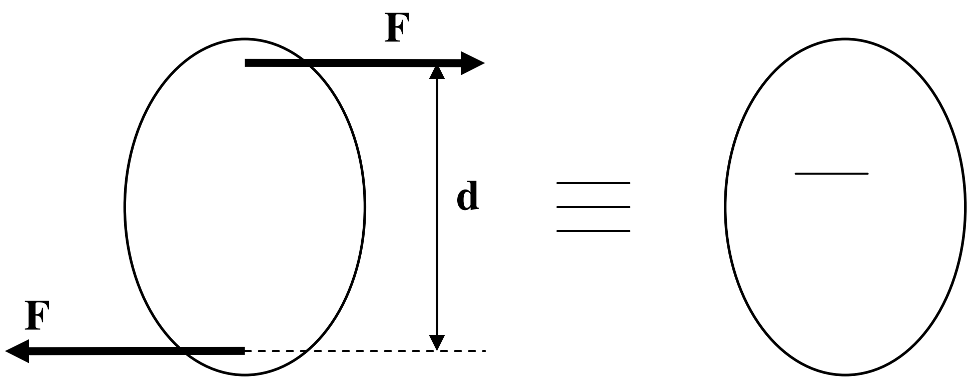


Figure 6 – a couple

Couples can be combined with forces to produce __________________________________.

Its main application is to translate the line of action of forces to a desired application point. In figure 7, a horizontal force F is applied at a distance *d* from point A. This force can be translated onto point A with an additional moment (couple) of magnitude *Fd*.

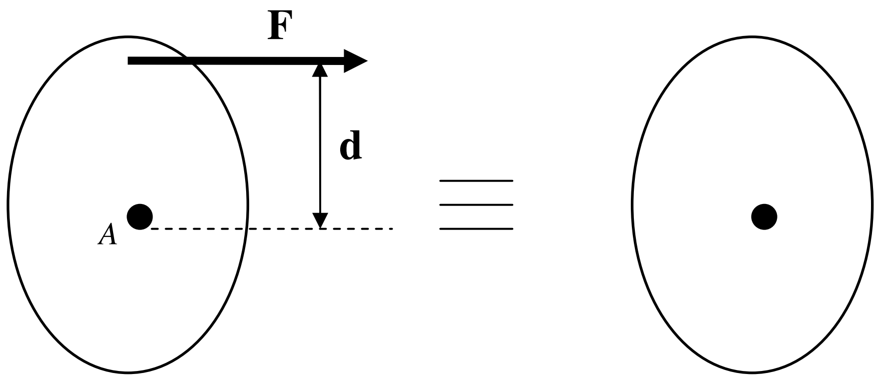


Figure 7

Similarly, a system where only a moment is present can also be represented by a pair of equal, opposite and noncollinear forces.

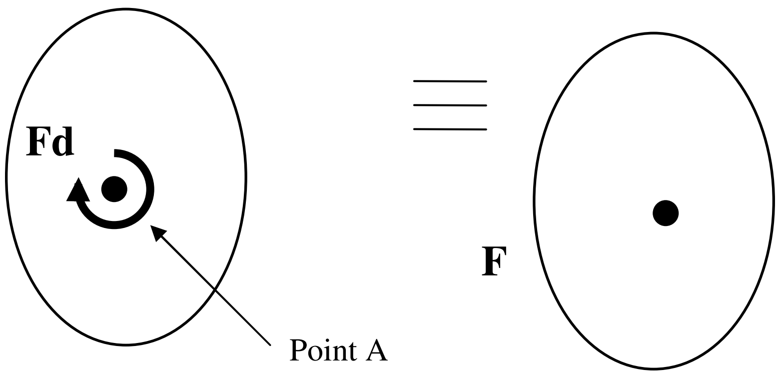


Figure 8

Part 2: Three-dimensional force systems

Systems of forces that appear in general applications are most likely three-dimensional, i.e. at least one force is not acting on the same plane. When this is the case, the three-dimensional vector approach must be used to analyse such a force system.

Vector system

First, it is very important to establish common background knowledge on the vector system. We will mainly use the _____________________________ coordinates here.

The Cartesian coordinates use three directional vectors to define the entire three-dimensional space, i.e. the unit vectors ____________________ are directional vectors in the _________________ direction _________________ axes, respectively.

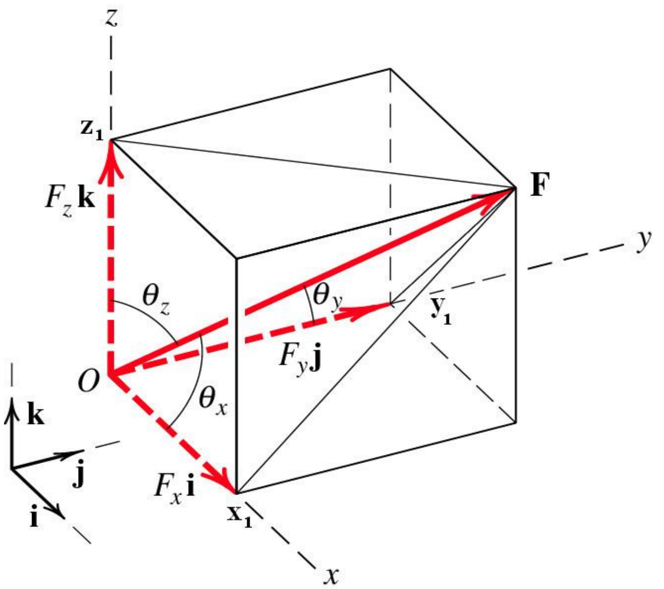


Figure 1

Figure 1 shows a force **F** in a 3-D space. Note that the angles θ_x, θ_y and θ_z are measured on the planes **OFx₁**, **OFx₂** and **OFx₃**, respectively. The properties of this force are given by

Magnitude of x-component of the force	
Magnitude of y-component of the force	
Magnitude of z-component of the force	
Magnitude of force **F**	
Force **F** defined in vector form	

Note: A unit vector is a vector whose magnitude is one.

Dot products of vectors

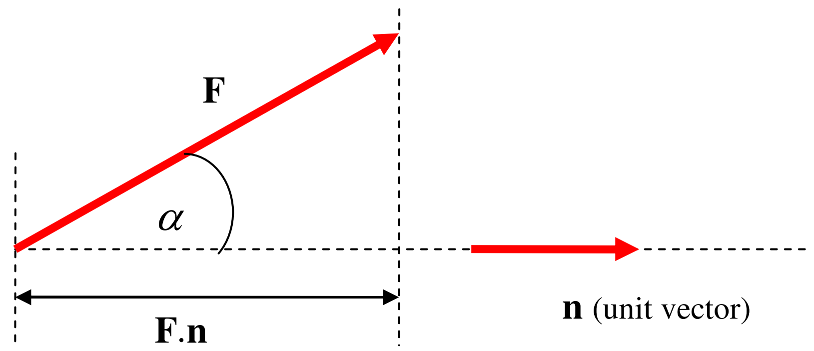


Figure 2

A force is represented by the vector **F**, which makes an angle α to the unit vector **n** as shown in figure 2. The dot product between the two vectors is ____________________ ____________________ and is given by

______________________________ *eqn. (1)*

The dot product is defined as the product of their magnitudes and the cosine of the angle between them. In a 3-D space, the angle is measured on the plane which contains both vectors.

Applications of dot products

1. It is used to determine the magnitude of a force in the direction of the unit vector. We can designate the unit vector to point towards any required direction.
2. The previous point can be further extended. As we already have the magnitude of the force in the direction of the unit vector, the force component in that direction may be given by

$$\mathbf{F}_n = (\mathbf{F} \cdot \mathbf{n})\mathbf{n} \qquad \textit{eqn. (2)}$$

3. It is used to determine the angle between the two vectors. By reversing equation (1), we obtain

$$\alpha = \cos^{-1}\left(\frac{\mathbf{F} \cdot \mathbf{n}}{Fn}\right) \qquad \textit{eqn. (3)}$$

Notes on dot products

Vectors **A** and **B** are in a 3-D space defined by the Cartesian coordinates. The unit vectors **i, j**, and **k** are in the positive directions of *x, y* and *z* axes, respectively. Given that the components of the two vectors are

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \quad \text{and} \quad \mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

Their dot product is given by

____

Cross products of vectors

Using the vectors **A** and **B** from the previous section, their cross product is defined as

$$\mathbf{A} \times \mathbf{B} =$$

Application of cross products

The cross product is used to determine the ________________________________

about an __________________ in the 3-D space.

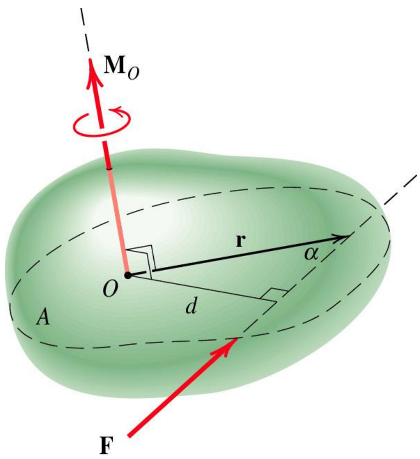


Figure 3

The line of action of force **F** in figure 3 is at a perpendicular distance *d* from point *O*, measured on the plane *A*.

The vector **r** connects point O with **any point** which lies on the line of action of force **F.** In this case **r** makes an angle α with the vector **F**.

The moment that force **F** generates about an axis perpendicular to plane A and point O is given by

______________________________ *eqn. (4)*

The __________________________ of the moment is determined by the right hand rule. By pointing the right hand thumb in the direction of the vector $\mathbf{M_0}$, the direction that the other fingers must bend in order to grasp the vector indicates the positive direction of the moment.

Equation (4) can also be used to determine _________________________________ in three dimensions. There must be a pair of ____________________ and ______________________ forces in space. Vector **r** is created to join any two points on the lines of action of the pair of forces. See figure 4 for illustration.

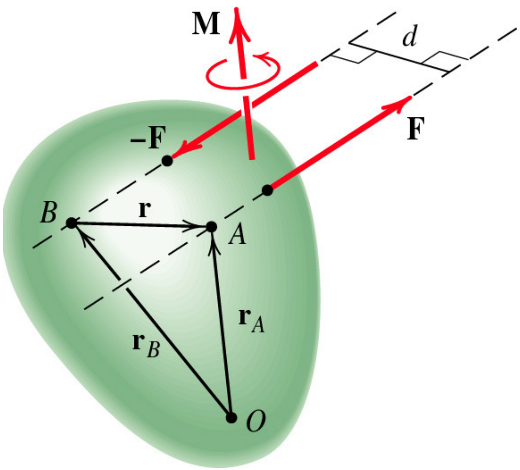


Figure 4

Moments about an arbitrary axis

Suppose that we require to compute the moment about an ______________________, which is __________________________________, i.e. not perpendicular to the plane which force **F** lies. This is achieved by creating a unit vector **n** in the direction of the required axis $\mathbf{M}_\lambda$. See illustration in figure 5. The moment about $\mathbf{M}_\lambda$ is given by

____

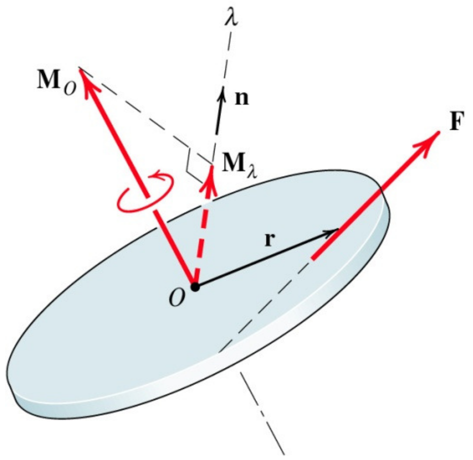


Figure 5

Wrench

When the resultant _________________________ and _______________________ vectors are ____________________ as shown in figure 6, the resultant is called a _____________________________.

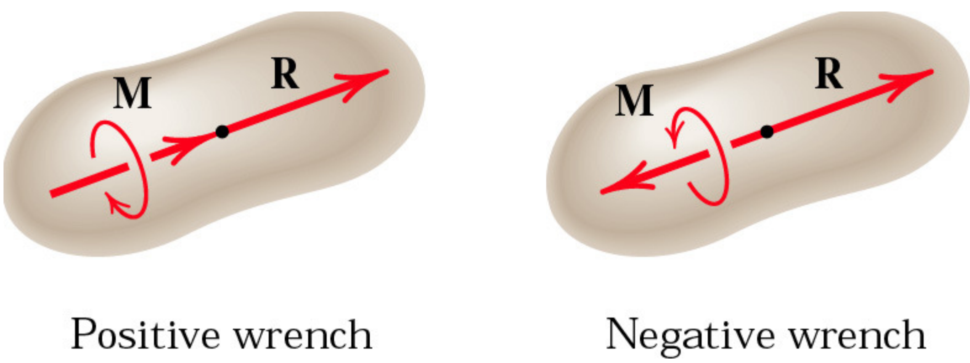


Figure 6